

VOLUME 2

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*Feynman*

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# LECTURES ON PHYSICS

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EXERCISES / 1964

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EXERCISES / 1964



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**CALIFORNIA INSTITUTE OF TECHNOLOGY**

**PASADENA, CALIFORNIA**

The set of exercises included here is a collection of problems given to Sophomores at Caltech during the years 1962-64, i.e., during the first two years that the course was revised using Professor Feynman's lectures in physics. The exercises were presented first either as homework exercises or as examination problems and thus vary greatly in difficulty. The order in which they are arranged here is roughly, but not at all strictly, in order of difficulty within each chapter. As with the exercises for Volume One, this set is not a "final" set and must be revised and added to as the course evolves.

The idea for about one-half the exercises was suggested by R. P. Feynman. The rest of the problems were contributed by those people teaching sophomore physics: J. Blue, T. Caughey, G. Chapline, M. Clauser, R. Dashen, R. Dolen, R. Griffith, F. Henyey, W. Karzas, R. Kavanagh, P. Peters, J. Pine, M. Plesset, M. Sands, I. Tammaru, A. Title, and C. H. Wilts.

A first editing of most of the problems was done by C. H. Wilts and myself after the 1962-63 school year. Although most problems are original, or at least original versions of "standard" problems, some problems were taken directly from: Introduction to Electricity and Optics, Second Edition, by N. H. Frank, McGraw-Hill 1950; and Physics for Students of Science and Engineering, by D. Halliday and R. Resnick, Wiley 1960. We thank the authors and publishers for permission to publish their problems.

The typing, both in the initial, generally hectic, stages, and in the final form was done by Mrs. F. L. Warren and is gratefully acknowledged.

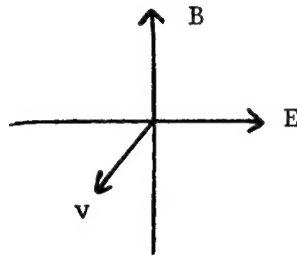
G. Neugebauer

There are no missing pages. Not all chapters had exercises.

## CHAPTER 1

- 1-1. Electric and gravitational forces.
- a) What would a proton's mass be if the gravitational force between two protons at rest were to equal the electric force? How does this compare with its actual mass?
  - b) What would be the electric force between two dimes (10¢) placed at opposite ends of a 10 meter lecture table if their nuclear and electronic charges were unbalanced by about 1%? Can you think of some object whose "weight" equals that force?
- 1-2. Make a rough estimate of the work that must be done against electric forces to assemble a uranium nucleus from two equal halves. What about assembling two deuterons to make a helium nucleus? Also express both answers in kilowatt hours per pound.
- 1-3. In copper there is one "conduction" electron for each copper atom. When a current of 10 amperes flows through a piece of No. 10 gauge copper wire, what is the average speed of the conduction electrons? How big is  $v^2/c^2$ ? (Remember that the ratio of "magnetic" effects to "electric" effects is about this small.)
- 1-4. In a certain region of space, there is a uniform electric field  $\vec{E}$  of 10,000 volts per centimeter in the +x direction. There is in the same region a uniform magnetic field  $\vec{B}$  in the +y direction. A beam of mu-mesons with the velocity  $c/3$  travels through this region on a straight line in the +z direction.

- a) What is the strength of the field  $\vec{B}$ ? (A mu-meson has a mass 210 times the electron mass and a charge equal in magnitude to the electronic charge.)
- b) Can you tell from this experiment if the charge on the mesons is + or - ?



1-5.

In the region of space of interest here there is a uniform magnetic field, such that  $B_x = 0$ ,  $B_y = 0$ , and  $B_z = B_0$ . The field is constant in time, and there are no currents or electric fields in the region of space we consider.

A particle of mass  $m$  and positive charge  $q$  is started at  $x = 0$ ,  $y = 0$ ,  $z = 0$  with a velocity  $v$  in the  $+x$  direction. Sketch and describe quantitatively in terms of  $B_0$ ,  $m$ ,  $v$ , and  $q$  the path of the particle. (Assume  $\frac{v}{c} \ll 1$ )

Suppose, that  $B_x = 0$ ,  $B_y = 0$ , but  $B_z = B_0 + ax$ . For  $(ax)$  always small compared to  $B_0$ , but not completely negligible, show on a sketch the qualitative behavior of the particle trajectory. (See Charpak, et al, Physical Review Letters, Vol. 6, 128 (1961) for the use of a similar field in an important experiment.) Show that the field just postulated is inconsistent with one of Maxwell's equations if the field fills a finite volume of space and, as above, you assume there are no currents or electric fields in the volume.

- 1-6. A particle with a mass  $\underline{m}$  and a positive charge  $\underline{q}$  is at a point  $x = z = 0$ ,  $y = a$ , and is moving with a low velocity

$$\vec{v} = v_0 \vec{e}_x$$

The charge is influenced by a negative charge  $-Q$  fixed at the origin and by a uniform magnetic field  $B_0$  in the  $+z$  direction. How large must  $B_0$  be such that the moving particle describes a circle of radius  $\underline{a}$  about the stationary one? If the magnitude of the magnetic field strength were different than this, explain why the speed of the particle is a function of the radial distance only.

Sketch qualitatively several cycles of the trajectory followed by the particle if it were released from the point  $x = z = 0$ ,  $y = a$  with zero velocity.



## CHAPTER 2

2-1. A copper wire of radius  $\underline{a}$  has a concentric insulating sheath with outer radius  $\underline{b}$ . The wire carries an electric current that raises its temperature to  $T_1$  while the outside of the insulation remains at  $T_2$  near room temperature.

- a) What is  $\vec{\nabla}T$  inside the insulation? Give answer in terms of  $\underline{a}$ ,  $\underline{b}$ ,  $T_1$  and  $T_2$ .
- b) How big is the temperature difference ( $T_1 - T_2$ ) if a current of 20 amps is sent through No. 10 gauge copper wire which is covered with a layer of rubber 0.2 cm thick whose thermal conductivity is  $1.6 \times 10^{-3}$  watt/cm<sup>0</sup>C?

2-2. Show by direct computation that

- a)  $\vec{\nabla} \cdot (\vec{\nabla} \times \vec{A}) = 0$
- b)  $\vec{\nabla} \times (\vec{\nabla} \times \vec{A}) = \vec{\nabla} (\vec{\nabla} \cdot \vec{A}) - \nabla^2 \vec{A}$

2-3. If  $\vec{R}$  is the vector from the origin to the point  $x, y, z$  show that

- a)  $\text{div } \vec{R} = 3$
  - b)  $\text{curl } \vec{R} = 0$
  - c)  $\vec{\nabla} \cdot (\vec{R}/R^3) = 0$
  - d)  $\vec{\nabla} \times (\vec{R}/R^3) = 0$
  - e)  $\vec{\nabla} (1/R) = -\vec{R}/R^3$
- } (Except at  $R = 0$ )
- f) From b) and Eq. (2.46) we know that  $\vec{R}$  can be written as  $\vec{R} = \vec{\nabla}\phi$ . What is  $\phi$ ?

2-4. Maxwell's equations are

- 1)  $\vec{\nabla} \cdot \vec{E} = \rho/\epsilon_0$
- 2)  $\vec{\nabla} \times \vec{E} = -\partial\vec{B}/\partial t$
- 3)  $\vec{\nabla} \cdot \vec{B} = 0$
- 4)  $c^2 \vec{\nabla} \times \vec{B} = \partial\vec{E}/\partial t + \vec{j}/\epsilon_0$

The conservation of charge can be written

$$5) \quad \vec{\nabla} \cdot \vec{j} = -\partial \rho / \partial t$$

- a) Show that 3) is consistent with the divergence of 2).
- b) Show that 5) follows from taking the divergence of 4) (i.e., the validity of Maxwell's equations requires that charge is conserved).
- c) Show that in empty space ( $\vec{j} = 0$ ,  $\rho = 0$ ),  $\vec{E}$  satisfies the wave equation

$$\nabla^2 \vec{E} - \frac{1}{c^2} \frac{\partial^2 \vec{E}}{\partial t^2} = 0$$

(Hint: take the curl of 2)).

- d) Show that in empty space,  $\vec{B}$  satisfies the same equation

$$\nabla^2 \vec{B} - \frac{1}{c^2} \frac{\partial^2 \vec{B}}{\partial t^2} = 0$$

- e) Show that 2) implies that  $\vec{E}$  can be written as

$$\vec{E} = -\vec{\nabla} \phi - \frac{\partial \vec{A}}{\partial t}$$

where  $\vec{A}$  is defined through  $\vec{B} = \vec{\nabla} \times \vec{A}$ . Why can  $\vec{B}$  be written in this way?

2-5. The velocity of a solid object rotating about an axis is a field  $\vec{v}(x, y, z)$ . Show that

- a)  $\vec{\nabla} \cdot \vec{v} = 0$
- b)  $\vec{\nabla} \times \vec{v} = 2\vec{\omega}$  where  $\vec{\omega}$  = angular velocity.

- 2-6. Prove by direct computation that if  $\vec{A}$  is a constant vector and  $\vec{R}$  is the radius vector, then

$$\text{curl } (\vec{A} \times \vec{R}) = 2\vec{A}$$

For vectors we know that

$$\vec{B} \times (\vec{A} \times \vec{C}) = \vec{A}(\vec{B} \cdot \vec{C}) - (\vec{B} \cdot \vec{A})\vec{C}$$

which might lead us to say that

$$\vec{\nabla} \times (\vec{A} \times \vec{R}) = \vec{A}(\vec{\nabla} \cdot \vec{R}) - (\vec{\nabla} \cdot \vec{A})\vec{R} = 3\vec{A} \quad (\text{False})$$

Why does the substitution of  $\vec{\nabla}$  for  $\vec{A}$  give an incorrect result?

- 2-7. A long steel shaft is subjected to a fancy heat treatment. At a particular time  $t$ , while it is cooling, the temperature distribution  $T(x)$  is as shown in Fig. 1. Isotherms for every  $10^\circ\text{C}$  interval are shown in Fig. 2. We assume throughout that the temperature depends only on  $x$ , the distance from one end of the bar. At the points A, B, and C draw arrows whose direction and magnitude are representative of the direction and magnitude of  $\vec{\nabla}T$ . At which of the five labeled points is the divergence of the heat flow  $\vec{h}$  largest? At how many of the five labeled points is  $\vec{\nabla} \times \vec{h} = 0$ ?

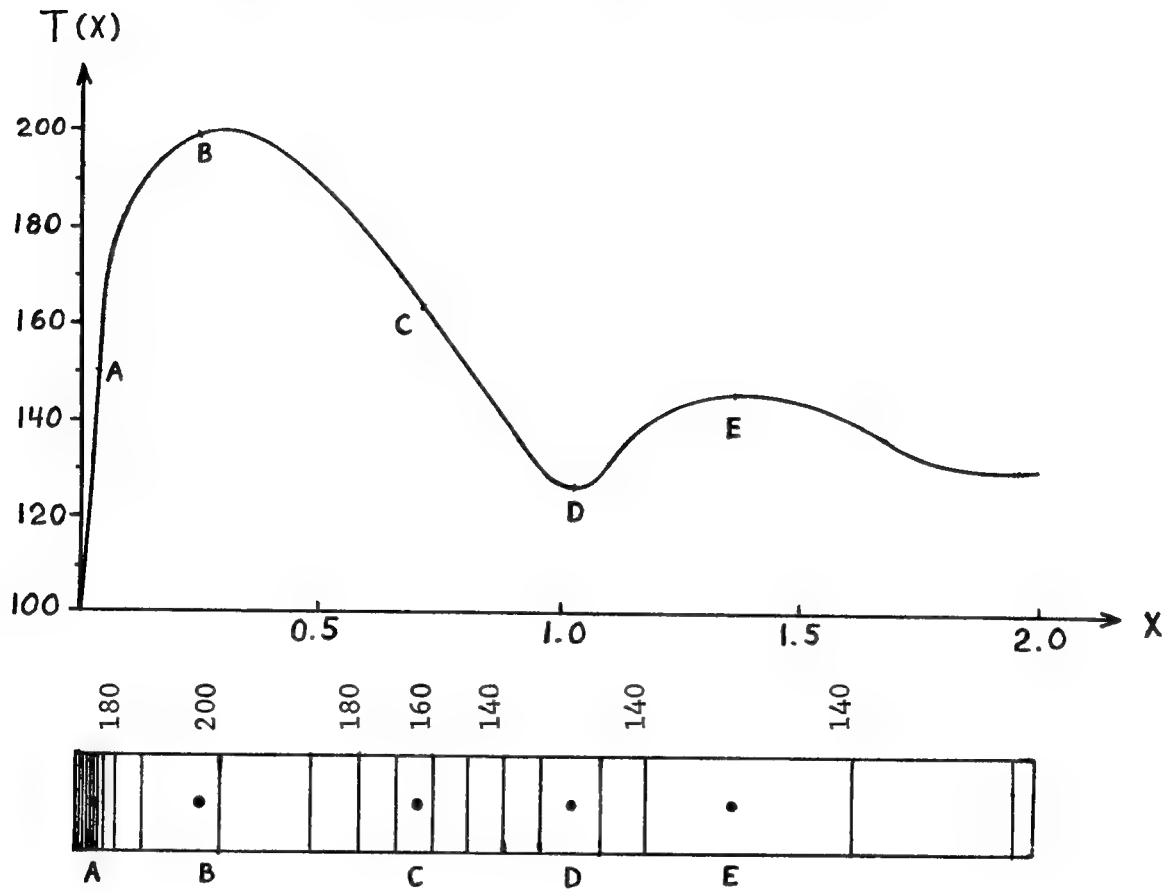
Fig. 1: Temperature ( $^{\circ}\text{C}$ ) vs. distance (m)

Fig. 2: Isotherms

### CHAPTER 3

- 3-1.           a) Maxwell's equations have been given in the form of a statement in Chapter 1 and in their differential form in Chapter 2. Show that the two forms are equivalent.
- b) Show that the equation

$$\vec{\nabla} \cdot \vec{j} = - \frac{\partial \rho}{\partial t}$$

is equivalent to a statement of the conservation of charge. In this equation  $\rho$  is the charge/unit volume and  $\vec{j}$  is the electric current density.

- 3-2.           A layer of radioactive material is deposited on the surface of a sphere. The material emits  $\alpha$  particles of high energy. Suppose that the particles are emitted only radially outward from the surface of the sphere. It would appear that this flow of charged particles constitutes a current. Is a magnetic field produced by this current?

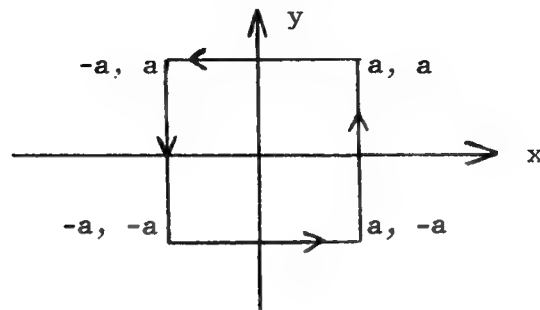
- 3-3.           The field of a point charge located at the origin is of the form

$$\vec{E} = \frac{K}{r^3} \vec{r}, \text{ where } r = (x^2 + y^2 + z^2)^{1/2}, \vec{r} = \vec{i}x + \vec{j}y + \vec{k}z,$$

and  $K$  is a constant.

- a) Calculate the flux of  $\vec{E}$  through the spherical surface of radius  $a$  with origin at the center.
- b) Use Gauss's theorem to connect the flux of  $\vec{E}$  through the spherical surface with the integral of  $\vec{\nabla} \cdot \vec{E}$  over the volume. Can you explain your result?

- c) Calculate the line integral of the vector  $\vec{E}$  around the path, in the x-y plane which is shown:

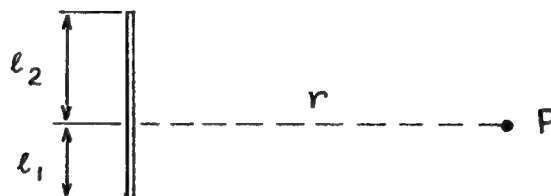


Use Stokes's theorem to verify the result.

- 3-4. Using the results of problem 3a, Chapter 2, find a (useless) formula for the volume of a region in terms of an integral over its surface. Check your result for a sphere and a rectangular block.

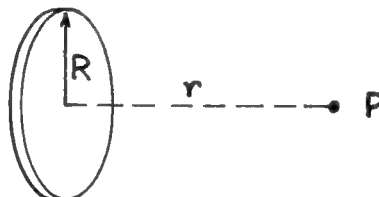
## CHAPTER 4

- 4-1. With reference to the figure below, find the potential  $\phi$  at the point P a distance  $r$  from a line of charge  $(\ell_1 + \ell_2)$  meters long which contains a charge density of  $\lambda$  coulombs/meter.



Compare your answer with the expected potential if  $r \gg (\ell_1 + \ell_2)$ . Check your answer in the limit  $r \ll (\ell_1 + \ell_2)$  by comparing the field derived from  $\phi$  with the field derived using Gauss's law.

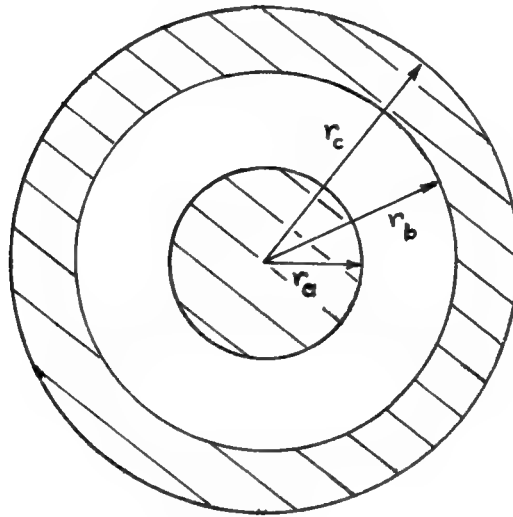
- 4-2. Calculate the electric field at the point P, a distance  $r$  from the center and on the axis of a thin uniformly charged disk of radius R.



- 4-3. A charge  $q'$  is placed on the inner of the two concentric metal spheres shown in the figure below; a charge  $q$  is placed on the outer sphere.

- a) Sketch the radial component of the electric field as a function of the radial distance.
- b) Sketch the potential relative to infinity as a function of the radial distance.

- c) What is the potential at the surface of the inner sphere?
- d) If the center sphere is moved off the center of the outer sphere, explain what happens to the field for  $r > r_c$  and for  $r_c > r > r_b$ .



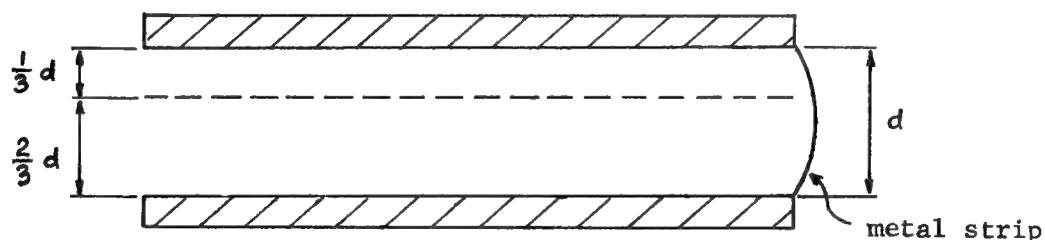


## CHAPTER 5

5-1. Show that the electric potential  $\varphi$  has the following interesting property: the average value of  $\varphi$  over an imaginary spherical surface is the same as the value of  $\varphi$  at the center of the sphere if there are no charges inside the sphere. Can you think of an application in which this property is useful?

5-2. Find the electric field inside but far from the ends of an extremely long cylinder which has a uniform charge density throughout. Note the difference between this result and the field inside a uniformly charged sphere.

5-3. Two large, flat metal plates are held parallel to each other and separated by a distance  $d$ . They are connected together at their edge by a metal strip. A thin plastic sheet carrying a surface charge  $\sigma$  per unit area is placed between the plates at a distance  $\frac{1}{3}d$  from the upper plate

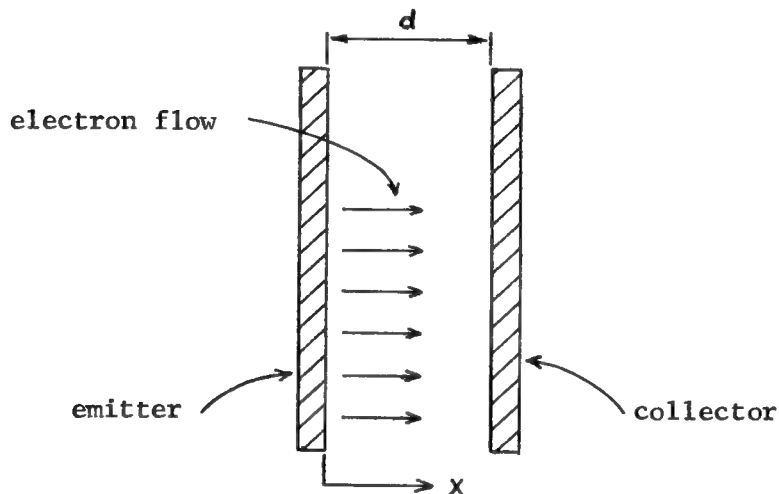


Call  $\vec{E}_1$  and  $\vec{E}_2$  the electric field near the upper and lower plates, respectively. What are  $\vec{E}_1$  and  $\vec{E}_2$ ?

5-4. Find the general formula for the  $\underline{x}$  component of the electric field if the charge density  $\rho$  varies only with  $\underline{x}$  throughout all space.

- 5-5. In a certain electronic tube, electrons are emitted from a hot plane metal surface, and collected by a plane metal plate parallel to the emitter, at the distance  $d$  away. (The distance  $d$  is small compared with the lateral dimensions of the plates.) The electric potential between the plates is given by  $\varphi = kx^{4/3}$  where  $x$  is the distance from the emitter.

- What is the surface charge density  $\sigma$  on the emitter?  
On the collector?
- What is the volume charge density  $\rho(x)$  for  $0 < x < d$ ?



- 5-6. Consider a conductor which has a charge distribution  $\sigma$  coulombs/m<sup>2</sup> on its surface;  $\sigma$  need not be a constant. Show that the force on the charge in a little element of area  $dA$  is normal to the surface and is given by  $(\sigma^2/2\epsilon_0)dA$ . (The factor of 1/2 is correct. Explain.)
- 5-7. The maximum field strengths that can exist at the surface of a conductor in a vacuum before field emission takes place is about  $10^8$  volts/meter. If the surface charge producing such a field is negative, compare the number of excess electrons per unit area to the number of atoms per unit area. Compare the force on an electron in this field to the force acting on an electron a typical atomic distance from a proton.

- 5-8. The negative muon ( $\mu$ -meson) is a particle that has the same electrical charge as the electron but a mass 207 times larger. When a negative muon is stopped in matter, it is attracted by a nucleus and may replace one of the atomic electrons and form a "mesic" atom. Because of its larger mass, the muon will approach closer to the nucleus than the electron and, for heavy nuclei, will even be inside the nucleus in the lowest energy state. The muon does not interact with nuclear matter through nuclear forces, but only through electrical forces and sees the nucleus as a sphere of uniform charge. From other experiments in nuclear physics it is found that the radius of nuclei can be given by  $R = R_0 A^{1/3}$  where  $R_0$  is approximately  $1.2 \times 10^{-15}$  meters and  $A$  is the number of protons plus neutrons.

Consider a model of the mesic atom where the muon oscillates back and forth along a line through the center of a lead nucleus. What is the natural frequency  $\omega$  of these oscillations? Recall from Chapter I-41 that the quantum energy states of a harmonic oscillator are separated by an energy difference  $\hbar\omega$ . What is the energy difference between the two lowest states of your model of the mesic atom? It is observed experimentally that when the mesic atoms are formed in lead 6 mev x-rays are emitted. How would you interpret this radiation?

- 5-9. Imagine that the earth were of uniform density and that a tunnel was drilled along a diameter. If an object were dropped into the tunnel show that it would oscillate with a period equal to the period of a satellite orbiting the earth just at the surface.
- 5-10. It is known that the earth gives off about  $8 \times 10^{20}$  joules per year in heat energy. Before studying thermal models in detail in order to explain this, it is useful to consider some models which are clearly over-simplifications but which do give order

of magnitude estimates. As an example, consider the possibility that the heat is all produced by radioactive materials uniformly distributed in the earth which decay giving off particles whose kinetic energy is converted into heat.

It is estimated that the temperature at the center of the earth is roughly  $2500^{\circ}\text{C}$  and that the thermal conductivity of typical materials is about  $0.03 \text{ joules/cm sec}^{\circ}\text{C}$ . Is the model described above consistent with these estimates?

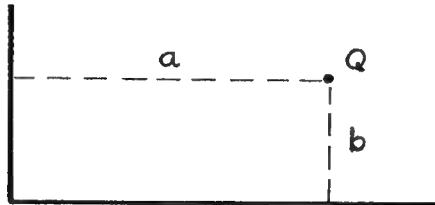
- 5-11. Two long concentric conducting cylinders are insulated from each other and charged. Far from the ends, the inner cylinder has a net charge density of  $+\lambda_1$ , and the outer one a net charge density of  $+\lambda_2$  coulombs per unit length. The inner cylinder has inner and outer radii  $r_1$  and  $r_2$ , while the outer has radii  $r_3$  and  $r_4$ . Find  $\vec{E}(r)$  at a point near the middle (i.e., where end effects can be neglected) and just outside the outside cylinder. Find the potential difference between the two cylinders.

Describe qualitatively any changes in the fields and potentials if:

- i.  $r_1$  is decreased
- ii.  $r_2$  is increased
- iii. the outside cross-section of the inner cylinder is made square with sides  $2r_2$  (assuming  $\sqrt{2} r_2 < r_3$ ).

## CHAPTER 6

- 6-1. Use the method of images to find the force on a charge  $Q$  located distances  $a$  and  $b$  from two semi-infinite conducting planes at right angles to each other.



- 6-2. A particle with electric charge  $q$  is released (from rest) at the distance  $x_0$  from the surface of a large, grounded, conducting plate. The particle is attracted by the plate, and moves toward it.
- What is the kinetic energy of the particle as a function of its distance  $x$  from the plate? (Neglect any energy loss by radiation.) Do you see anything unphysical about your answer?
  - A real metal plate can be thought of as an ideal conducting plane only for distances down to the atomic spacing, that is, to about  $1 \text{ \AA}$ . Estimate the kinetic energy with which an electron would arrive at a conducting plate, if released at rest 1 cm from the plate. Give your answer in electron-volts.
- 6-3. A rectangular box of insulating plastic 1 cm by 10 cm by 100 cm is filled with a uniform charge density  $\rho$  coulombs per  $\text{cm}^3$ . Consider a straight line that is perpendicular to the 10 cm by 100 cm face and passes through the center of the box. Sketch roughly a graph of the potential  $\phi$  along this line as a function of the distance from the center. Consider the range

of distances from 0.001 cm (i.e., inside the box) to distances much larger than 100 cm; a  $\log(\varphi)$  vs  $\log(\text{distance})$  plot is appropriate. On the same graph sketch a curve that gives the magnitude of the electric field  $\vec{E}$ .

- 6-4. The earth is continually bombarded by high energy cosmic rays that come from outside the solar system. It has been determined by high altitude measurements from balloons and satellites that the cosmic rays are made up almost completely of protons, although there are also a few percent alpha-particles, heavier nuclei, and electrons. The mean energy of the bombarding protons is a few billion electron volts; the intensity of the protons arriving at the earth's atmosphere is about one proton per  $\text{cm}^2$  per sec.

One might wonder how long it would take for the charge arriving at the earth in cosmic rays to raise the earth's potential to the point that the protons could no longer reach the earth. How does the time required to accomplish this compare to the approximate 5 billion year age of the earth? If the time is shorter than the earth's age, one is faced with the question: "Why are cosmic rays still coming?"

- 6-5. Find the capacitance per unit length of a long cylindrical capacitor made up of a conducting cylinder of radius  $a$  inside and co-axial with a cylinder of radius  $b$ . Qualitatively, what would happen if there were imperfections in the construction which, in effect, inserted a sharp protrusion from the outer wall?

- 6-6. a) What is the surface charge density induced on an uncharged, insulated, conducting sphere located a

distance b from a point charge  $+q$ ?

- b) If the sphere is raised to a potential  $V$ , what is the force between the charge and the sphere?

6-7. In Chapter 6 it is demonstrated qualitatively that it is possible to find the field outside a sphere with a surface charge density varying as  $\cos \theta$  superposing the fields of two slightly offset oppositely charged spheres. Carry out the process quantitatively and find the field both outside and inside a sphere with a surface charge density  $\sigma(\theta) = A \cos \theta$  where  $A$  is a constant and  $\theta$  is the polar angle.

6-8. The field of a dipole is given in Eqs. 6.14 and 6.15.

a) Find the radial and tangential components of the field at a point  $(r, \theta, \varphi)$ .

b) Show that the electric field of a dipole points in the same direction at all points on any given straight line which passes through the dipole.

c) At any given distance from the dipole, what is the direction and relative strength of  $\vec{E}$  for points lying at angles of  $0$ ,  $\pi/4$ , and  $\pi/2$  from the direction of  $\vec{p}$ ?

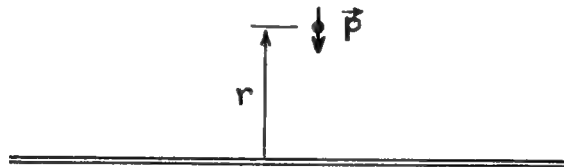
6-9. Consider a dipole placed in an electric field which, before the dipole was inserted, was uniform and of strength  $\vec{E}_0$ .

a) If the dipole moment points in the direction of the external field there will be an equipotential surface which encloses the dipole. Show that this surface is a sphere, and find the strength of the dipole moment for which the sphere will have radius a.

- b) Sketch the electric field outside the sphere.
- c) If a thin conducting shell at the same potential were made coincident with the equipotential surface, how would the fields be changed?
- d) What would be the charge density on the sphere?
- e) What would be the dipole moment of this charge density?
- f) How would you utilize the results of the above?

6-10. A particle with an electric dipole moment  $\vec{p}$  is placed at the distance  $r$  from a long wire which has a charge  $\lambda$  per unit length ( $\lambda$  is a constant). The dipole moment lies in the plane defined by the wire and the particle.

- a) What is the force and torque on the particle if  $\vec{p}$  is normal to the wire?
- b) What is the force and torque if  $\vec{p}$  is parallel to the wire?

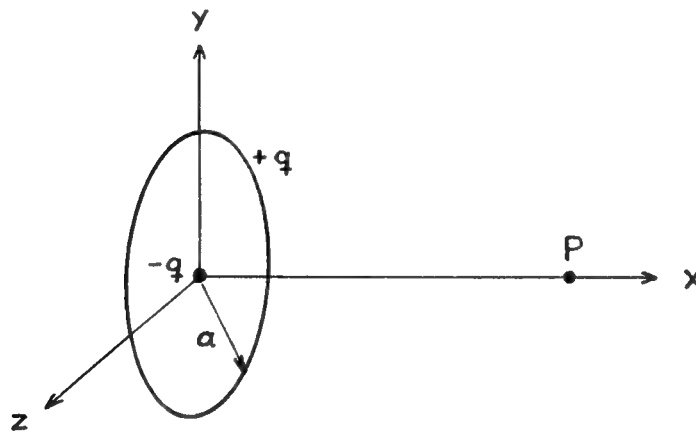


6-11. Find the potential as a function of distance away from a very large sheet of dipoles. Assume that there are  $\Sigma$  dipoles per unit area and that each dipole has dipole moment  $\vec{p}$  and is pointing normal to the surface.

6-12. An electric charge  $+q$  is distributed uniformly on a thin ring of radius  $a$ . The ring is placed in the  $y$ - $z$  plane with its center at the origin. A charge  $-q$  is placed at the origin.

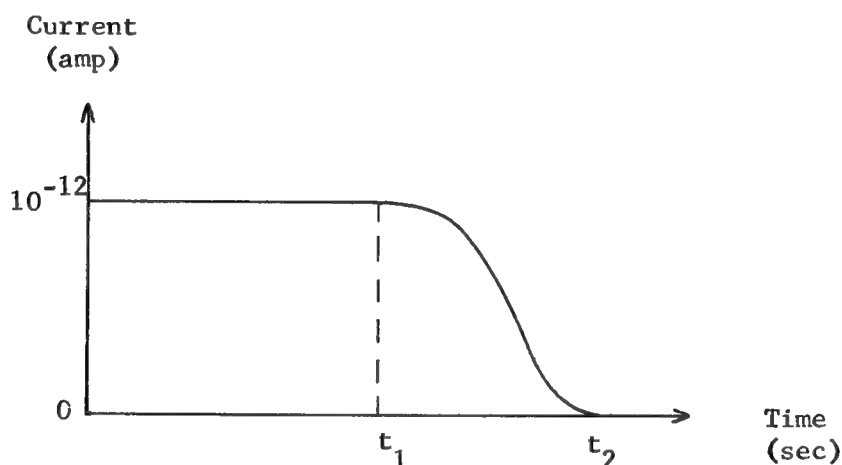


- a) Find the potential  $\varphi$  at the point P on the x-axis, at a distance  $\underline{x}$  from the origin.
- b) What is the electric field at this point?
- c) How does the electric field vary with  $\underline{x}$  for  $x \gg a$ ? How does the decrease at large distances compare with the field of a dipole? Can you explain?



- 6-13. A parallel plate capacitor with a capacitance of  $100 \mu\text{f}$  and a separation of 1 cm is charged with a battery to a potential difference of 10 volts. The battery is then disconnected from the capacitor. Blue light is shined on the bottom plate, which causes electrons with kinetic energies ranging from 0 to 1.5 eV to be emitted. The battery voltage is such that the electrons are attracted to the upper plate. The total current which goes to the upper plate is shown in the figure as a function of time. How long will it take for the potential difference between the plates to become zero (time  $t_1$  in the figure)? What is the potential difference for time very much larger than  $t_2$ ?

If the plate separation was doubled before the capacitor was charged, how would your answer change? If the plate separation were doubled after the capacitor was fully charged and after the battery was removed, how would your answer change?



- 6-14. An insulating rod 1 meter long and 1 centimeter in radius has its axis along the x-axis and its ends at  $x = -0.5$  meter and  $x = +0.5$  meter. It has a total volume charge density given by  $\rho = ar^2$ , where  $r$  is the distance to the axis of the rod and  $a$  is a positive constant,  $2 \text{ coulombs/m}^{-5}$ .
- Find the electric field  $E$  at the four points  $x = 0$ ;  $z = 0$ ;  $y = 0, 0.5, 1.0$ , and  $2.0$  cm. In this part assume the rod is infinite in length.
  - Make a good estimate of the potential at  $x = 0$ ,  $y = 0$ ,  $z = 0$ , relative to zero potential at infinity. Assign an uncertainty to your estimate and justify it.
  - Will the potential at  $x = 0.5$  meter,  $y = z = 0$  be greater than, less than, or equal to the potential at the origin?

## CHAPTER 7

- 7-1.
- a) Show that the equipotentials produced by two parallel line charges of opposite sign are circular cylinders. Assume a density of  $+\lambda$  and  $-\lambda$  coulombs/meter on two lines separated by a distance  $\underline{d}$ .
  - b) From the above result find the capacitance per unit length of two parallel wires of radius  $\underline{r}$  with their centers separated a distance  $\underline{d}$ . Assume that  $d \gg 2r$ .
  - c) Show that if  $x, y \gg d$  the potential can be obtained from the complex function

$$f(z) \equiv U + iV = \frac{1}{z} = \frac{1}{x + iy}$$

## CHAPTER 8

- 8-1. Do you agree with the statement made in Chapter 8 that a nucleus which contains  $Z$  protons distributed more or less uniformly through the volume of a sphere of radius  $\underline{r}$  has an electrostatic energy of about the value given by

$$U = \frac{3}{5} Z (Z - 1) \frac{e^2}{r} \quad . ?$$

- 8-2. A radio tuning capacitor has a maximum capacity of 100 pf (1 pf =  $10^{-12}$  f). By rotation of the moving plates, the capacity can be reduced to 10 pf. Assume the capacitor is charged to a potential difference of 300 volts at maximum capacity. The tuning knob is then rotated to minimum capacity. What are the initial and final values of the potential difference? How much mechanical work is done in rotating the knob?
- 8-3. Two capacitors with capacitance  $C_1$  and  $C_2$  are initially charged with charges  $q_1$  and  $q_2$ . Show that except in special cases the stored electrostatic energy decreases when the two capacitors are joined together in parallel. Where does the lost energy appear? Find the conditions under which they can be joined without loss of energy.
- 8-4. Show that when a dipole of dipole moment  $\vec{p}$  is placed in an electric field  $\vec{E}$ , the electrostatic energy is given by:

$$U = - \vec{p} \cdot \vec{E}$$

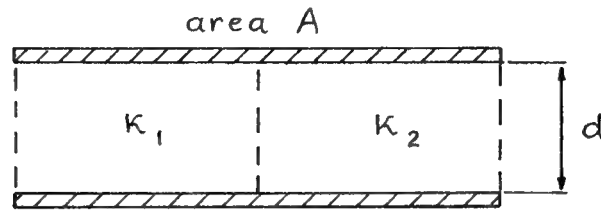
Calculate the torque exerted on the dipole by the field  $\vec{E}$ .

Do the calculation directly as well as by using the above energy relation. Is the energy the same if the dipole were formed from two charges placed sequentially in the field? If not, calculate the difference; if so, justify your reasoning physically.

- 8-5. Show that the plates of a parallel-plate capacitor attract each other with a force given by  $F = \frac{q^2}{2\epsilon_0 A}$ . Consider the work necessary to increase the plate separation from  $\underline{x}$  to  $(x + dx)$ .
- 8-6. The pi meson (or pion) is a particle which is found in all three charged states: i.e., there are positive, negative and neutral pi mesons. The mass (times  $c^2$ ) of charged pions is 139.6 Mev while the mass of neutral pi mesons is 135.0 Mev. In one model of the pion, the mass difference is assumed to be caused only by the electrostatic energy. If one further assumes that pions are represented as spheres and that the charge of charged pions is uniformly distributed throughout this sphere, it is possible to calculate the "radius" of the pion. Under these assumptions calculate the radius of the pion. Is your result compatible with other estimates of nuclear dimensions?
- 8-7. A metal spherical shell of inner radius  $\underline{a}$  and outer radius  $\underline{b}$  is located with its center at the origin. There is a small hole cut at one point of the shell. If there is no net charge on the shell, how much work is required to bring a charge  $q_1$  from infinity, through the hole, and to the origin. How much is required if the shell is given a total charge  $q_2$ ?

## CHAPTER 10

- 10-1. A parallel-plate capacitor is filled as shown below with two dielectrics of equal size but unequal  $\kappa$ .



Show that the capacitance is given by  $C = \frac{\epsilon_0 A}{d} \frac{\kappa_1 + \kappa_2}{2}$ .

- 10-2. A parallel plate capacitor has square plates 20 cm on a side and a plate separation of 1 cm. It is charged to a potential difference of 10 volts and then disconnected from the battery. A large square sheet of dielectric slightly less than 1 cm thick is inserted between the plates so that a 10 by 20 cm area of the capacitor is filled with dielectric. The dielectric constant is 4.0.



- a) What is the attractive force between the plates?
- b) What is the dipole moment per unit volume in the dielectric at positions well inside the plates (so that edge effects can be neglected)?
- c) Suppose the dielectric sheet consisted of material of dielectric constant 4.0 which had embedded in it a uniform distribution of small metal spheres. Would the potential difference between the plates be greater than, less than, or equal to, that for the original homogeneous dielectric?

- 10-3. A parallel-plate capacitor of separation  $\underline{d}$  has a capacitance  $C_0$  in air. An insulating slab of dielectric constant  $\kappa$ , thickness  $t < d$ , and area equal to that of the plates is inserted into the capacitor, the slab faces being parallel to those of the capacitor. Neglecting end effects, prove that the capacitance is now

$$C = \frac{C_0}{1 - [(\kappa - 1)/\kappa] (t/d)} \quad .$$

- 10-4. An isolated metal sphere of radius  $\underline{a}$  has a free charge  $Q$  on its surface. The sphere is covered with a dielectric layer with inner radius  $\underline{a}$  and outer radius  $\underline{b}$ .
- Calculate the polarization surface charge on the inside and outside of the dielectric.
  - What is the volume density of polarization charge inside the dielectric?

- 10-5. A parallel-plate capacitor is connected to a battery which maintains a potential difference  $V_0$  between its plates. A slab of dielectric constant  $\kappa$  is inserted between the plates, completely filling the space between them.
- Show that the battery does an amount of work  $q_0 V_0 (\kappa - 1)$  during the insertion process, if  $q_0$  is the charge on the capacitor plates before the slab is inserted.
  - How much work is done by mechanical forces on the slab when it is inserted between the plates? Is this work done on, or by, the agent inserting the slab?

- 10-6. Two coaxial pipes of radii  $\underline{a}$  and  $\underline{b}$  ( $a < b$ ) are lowered vertically

into an oil bath. If a voltage  $V$  is applied between the pipes, show that the oil rises a height

$$H = \frac{V^2 (\kappa - 1) \epsilon_0}{\ln \left( \frac{b}{a} \right) \rho (b^2 - a^2) g}$$

where  $\kappa$  is the dielectric constant of the oil.

- 10-7. Show that, when a line of force cuts through a surface separating two dielectrics of dielectric constants  $\kappa_1$  and  $\kappa_2$ , it makes angles  $\theta_1$  and  $\theta_2$  with the normal to the surface in the two media, given by the relation  $\kappa_1 \cot \theta = \kappa_2 \cot \theta_2$ .

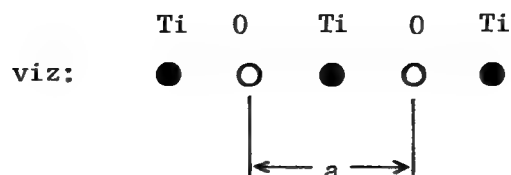


## CHAPTER 11

- 11-1. Consider a sphere of dielectric material with radius  $a$  which is uniformly polarized (see Fig. 11-7). Find the electric field inside and outside this sphere.
- 11-2. The dielectric constant of helium at  $0^{\circ}\text{C}$  and 1 atm pressure is 1.000074. Find the dipole moment induced in each helium atom when the gas is in an electric field of intensity  $10^2$  volts/m.
- 11-3. Water vapor is a polar gas whose dielectric constant exhibits an appreciable temperature dependence. The following table gives experimental data on this effect. Assuming that water vapor obeys the ideal gas law, calculate the molecular polarizability as a function of inverse temperature and plot it. From the slope of the curve, deduce a value for the permanent dipole moment of the  $\text{H}_2\text{O}$  molecule.

<u>T(<math>^{\circ}\text{K}</math>)</u>	<u>Pressure (cm Hg)</u>	<u><math>(\kappa - 1) \times 10^5</math></u>
393	56.49	400.2
423	60.93	371.7
453	65.34	348.8
483	69.75	328.7

- 11-4. Consider a system consisting of 2 atoms separated by a fixed distance  $a$ , each atom having polarizability  $\alpha$ . Find the relation between  $a$  and  $\alpha$  for such a system to be ferroelectric. (Answer:  $a^3 = \alpha/2\pi$ ). If you find this too easy, consider a line of oxygen atoms, regularly spaced with a distance  $a$  between each atom. Suppose also that there is a titanium atom half-way in between successive oxygen atoms.



Let the polarizability of oxygen be  $\alpha_O$ , titanium  $\alpha_T$ . Find the conditions on  $\alpha_O$  and  $\alpha_T$  such that the system is a ferroelectric.

Note: 
$$\sum_{n=1}^{\infty} \frac{1}{n^3} = 1.20 ,$$

$$\sum_{n=1}^{\infty} \frac{1}{(2n)^3} = \frac{1}{8} \sum_{n=1}^{\infty} \frac{1}{n^3} .$$

- 11-5. A "dielectric" material consists of a number of brass spheres of diameter  $d$ , spaced  $3d$  apart, in a regular lattice. Assuming that each sphere is influenced only by the imposed external electric field (i.e., neglecting the effect of neighboring spheres or the redistribution of induced charges), find the dielectric constant  $\kappa$  for this material.

## CHAPTER 12

- 12-1. A ring of radius  $\underline{a}$  is made of copper wire whose diameter  $\underline{b}$  is much less than  $\underline{a}$ . The ring is imbedded in the center of a plastic sphere whose radius is very large compared with the radius  $\underline{a}$  of the ring. An alternating magnetic field induces a current in the ring. The current heats the wire, generating thermal energy at the rate  $W$  (joules per second). The temperature at the outside surface of the large sphere is  $T_0$ . In the steady state, what is the temperature at the center of the ring?
- 12-2. In the problem set for Chapter 5, one simple thermal model of the earth was considered. Another (again over-simplified) model is that inside the earth there is a core of extremely high conductivity and of radius  $\underline{a}$ . Find how large this core must be if its temperature is  $2500^\circ\text{C}$ , if the thermal conductivity of the surrounding earth is  $0.03 \text{ joules/cm-sec-}^\circ\text{C}$ , and if  $8 \times 10^{20}$  joules of heat energy are given off by the earth each year.
- 12-3. a) For certain geometries and physical conditions, it is useful to write the potential in the form  $\varphi = f(r) \cos \theta = f(r) \frac{z}{r}$  where  $r^2 = x^2 + y^2 + z^2$ . (This form was used, for example, in Chapter 12 to solve the problem of the flow of "dry water" around a sphere.) If  $f(r)$  is expanded in a power series

$$f(r) = \sum_{n=-\infty}^{\infty} b_n r^n$$

only two of the coefficients  $b_n$  can be non-zero if  $\varphi$  is to satisfy Laplace's equation. Find these two.

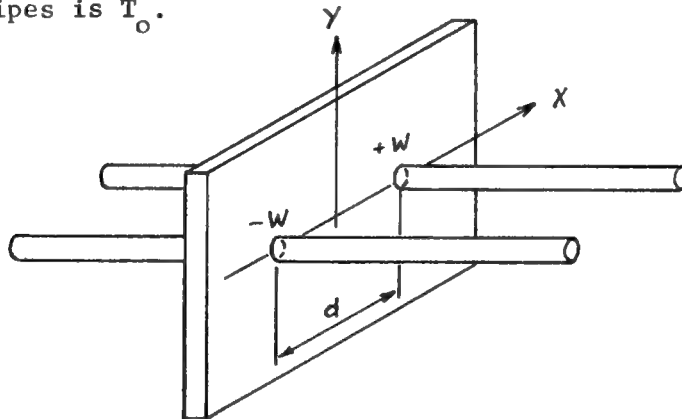
- b) For the analogous two dimensional problems, the potential may be written in the form

$$\varphi = g(\rho) \cos \theta = g(\rho) \left(\frac{z}{\rho}\right); \quad g(\rho) = \sum_{n=-\infty}^{\infty} c_n \rho^n,$$

where  $\rho^2 = y^2 + z^2$ . Find which values of  $c_n$  can be non-zero if  $\varphi$  is to satisfy Laplace's equation.

Use cartesian coordinates to do your calculations.

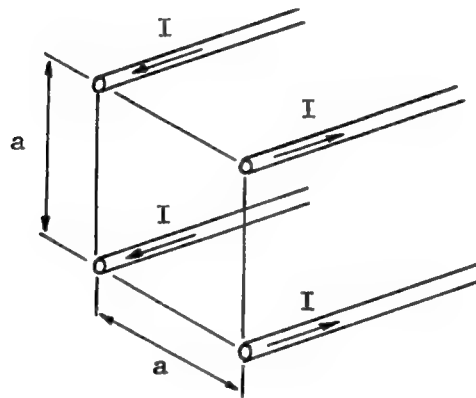
- 12-4. Two thin water pipes are separated by a distance  $d$  and go normal to and through a large wall of thickness  $t$ . The thermal conductivity of the wall is  $K$  while the temperature far from the pipes is  $T_0$ .



Hot water which gives off  $+W$  watts to the wall flows in the pipe at  $x = +d/2$  while cold water which absorbs  $W$  watts from the wall flows in the other pipe. Consider this as a two dimensional problem and neglect the finite size of the pipes. Find the temperature at a point  $P$  located at  $x = 100d$ ,  $y = 100d$ . Let  $T_0 = 20^\circ\text{C}$ ,  $d = 50\text{ cm}$ ,  $k = .03\text{ watts/cm}^\circ\text{C}$ ,  $W = 200\text{ watts}$  and the wall thickness be  $10\text{ cm}$ . Make suitable approximations in evaluating your answer.

### CHAPTER 13

- 13-1. Four long No. 10 copper wires are parallel to each other, their cross section forming a square 20 cm on edge. A 20 amp current is set up in each wire in the direction shown in the figure below. What are the magnitude and direction of  $\vec{B}$  at the center of the square? What is the magnitude and direction of force per meter acting on the lower left wire?



$$a = 20 \text{ cm}$$

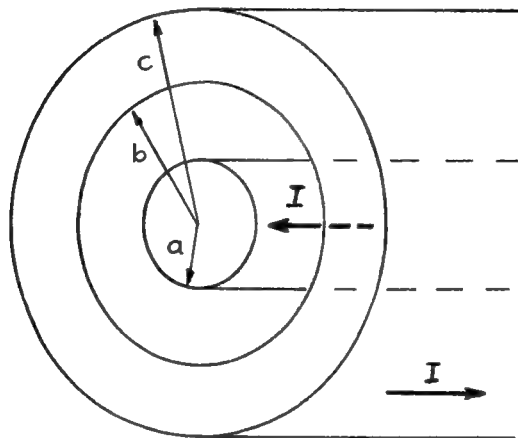
$$I = 20 \text{ amp}$$

- 13-2. A long, solid dielectric cylinder of radius  $a$  is permanently polarized so that the polarization is everywhere radially outward, with a magnitude proportional to the distance from the axis of the cylinder, i.e.,  $\vec{P} = P_0 \vec{r}/2$ . The cylinder is rotated with constant angular velocity  $\omega$  about its axis. What is the magnetic field on the axis of the cylinder, at points not too close to the ends?

13-3.

A long coaxial cable consists of two concentric conductors with the dimensions shown in the figure below. There are equal and opposite currents  $I$  in the conductors. It may be assumed that the currents are uniformly distributed in the conductors.

- Find the magnetic field  $\vec{B}$  at  $\underline{r}$  within the conductor ( $r < a$ ).
- Find  $\vec{B}$  between the two conductors ( $a < r < b$ ).
- Find  $\vec{B}$  within the outer conductor ( $b < r < c$ ).
- Find  $\vec{B}$  outside the cable ( $r > c$ ).

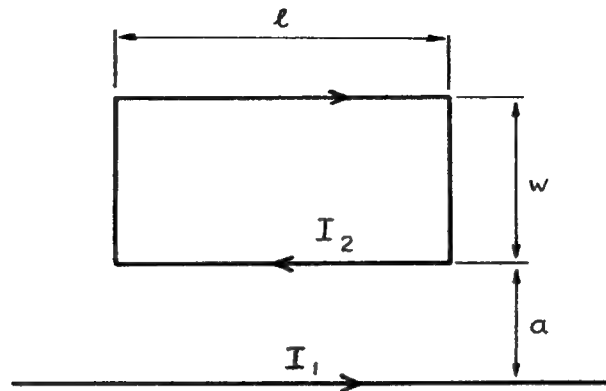


13-4.

A long wire carries the current  $I_1$ , while a rectangular loop of wire whose length and width are  $\underline{\ell}$  and  $\underline{w}$  respectively carries the current  $I_2$ . The wire lies in the plane of the loop.

- What is the force on the loop? What is the force on the long wire?

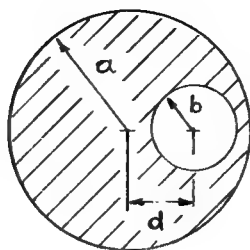
- b) What is the torque on the loop? What is the torque on the wire?



- 13-5. The scan of a TV picture tube is generated by deflecting the electron beam with a magnetic field which comes from a set of deflecting coils that are attached around part of the neck of the picture tube. The electrons are typically emitted from the gun (before deflection) with an energy of three keV; they are further accelerated after deflection.

Estimate the magnetic field at the electron beam at the time of maximum deflection. Also estimate the number of ampere-turns in the coil at the same time. You may neglect the post deflection acceleration in making your estimate. In what direction would your answer change if you were to include it?

- 13-6. A very long conducting rod of radius  $a$  has an off-center hole of radius  $b$  whose axis is parallel to but off-set by a distance  $d$  from the axis of the rod.



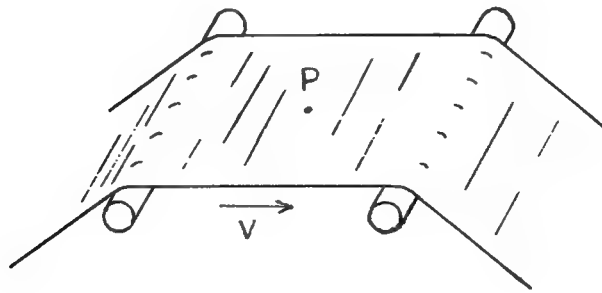
A uniform current density  $+j$  flows in the conductor. What is the magnetic field  $B$  at the axis of the hole, far from the ends?



## CHAPTER 14

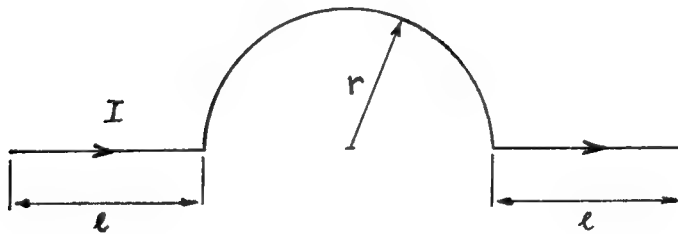
14-1. In a plastic film factory, a wide belt of thin plastic material is travelling between two successive rollers with the speed  $\underline{v}$ . In the manufacturing process, the film has accumulated a uniform surface electric charge ( $\sigma$ ).

- What is the vector potential near the surface of the belt in the middle of a large flat span? (Near point P of the figure.)
- What is  $\vec{B}$  in the same region?

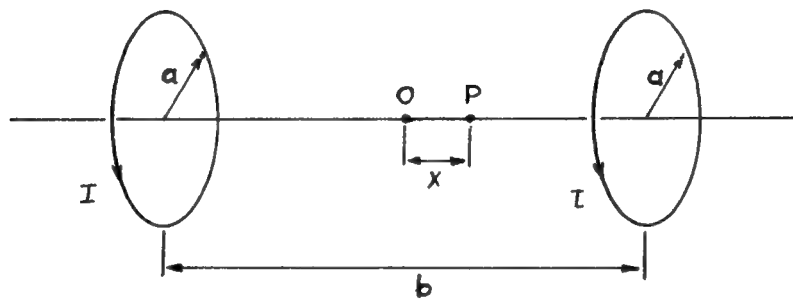


14-2. A wire of the shape shown in the figure carries a current  $I$ . What is the magnetic field at the center of the semicircle arising from

- each straight segment of length  $\ell$ ?
- the semicircular segment of length  $\pi r$ ?
- the entire wire?



- 14-3. In practical magnetic structures, uniform magnetic fields are frequently necessary. The uniformity of the field produced by Helmholtz coils, or two co-axial loops which carry currents in the same direction, is one of their most important characteristics. Assume that the coils have a radius  $\underline{a}$ , have axes on the x-axis, carry a current  $I$  each, and are separated by a distance  $\underline{b}$ .



- Find the magnetic field at a point  $P$  on the axis of the loops and a distance  $\underline{x}$  from the mid-point  $O$ .
- Expand the expression for the field in a power series retaining terms to order  $x^2$ .
- What relationship must exist between  $\underline{a}$  and  $\underline{b}$  such that the  $x^2$  terms vanish? What is the significance of this?
- Show that the field created by the coils to this order and under the condition established in c) is given by

$$B_x = \frac{8I}{5^{3/2} a \epsilon_0 c^2}$$

14-4. A square loop of wire of edge a carries a current I.

- a) Using the Biot-Savart law show that the magnetic field  $\vec{B}$  for a point on the axis of the loop and a distance x from its center is given by:

$$B = \frac{4a^2 I}{\pi(4x^2 + a^2)^{3/2} \epsilon_0 c^2}$$

- b) Find the same result using the vector potential.  
Hint: Leave the vector potential in its integral form and use:

$$\frac{\partial}{\partial x} \int_a^b f(x,y) dy = \int_a^b \frac{\partial}{\partial x} f(x,y) dy$$

$$\lim_{x \rightarrow c} \int f(x,y) dy = \int \left\{ \lim_{x \rightarrow c} f(x,y) \right\} dy$$

14-5. Use the vector potential to calculate the magnetic field at any point on the axis of a circular loop of radius a which carries a current I.

14-6. Consider a conducting sphere of radius a which is raised to a potential V. If the sphere is rotated about one axis with an angular velocity  $\omega$ , show that the magnetic field outside corresponds to that of a dipole field with a moment

$$\mu = \epsilon_0 \omega V \left( \frac{4}{3} \pi a^3 \right)$$

Show also that the field inside is given by

$$B_{\text{int}} = \frac{2\omega V}{3c^2}$$

These results are exact everywhere inside and outside.  
 If you cannot show this, derive the first result for distances much larger than  $a$ , and the second result at the center.

- 14-7. It has been pointed out that the rotation of the earth could possibly be measured by measuring the potential difference between the center and edge of a charged conducting cylinder placed at the North Pole with its axis pointing to the center of the earth.

Show that the potential difference can be written in the form

$$V = \left(\frac{v}{c}\right)^2 \frac{\lambda}{4\pi\epsilon_0}$$

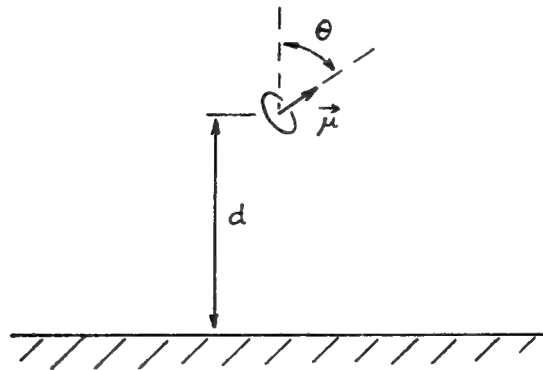
where  $v$  is the velocity of a point at the outside edge of the cylinder and  $\lambda$  is the surface charge per unit length. See if you can visualize reasonable parameters of the equipment necessary to measure this effect.

## CHAPTER 15

15-1.

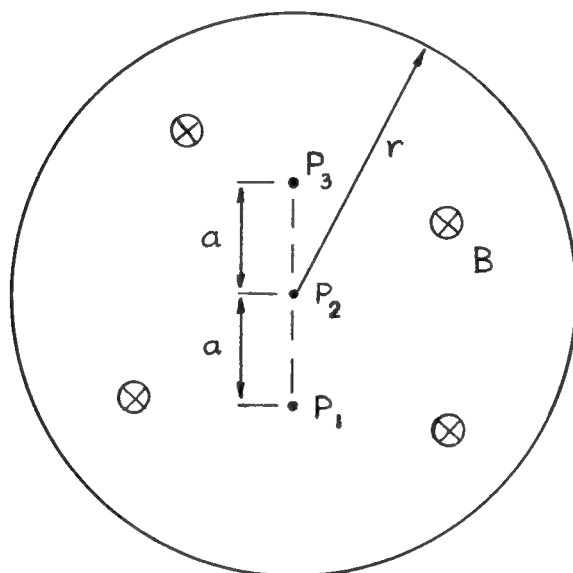
A superconducting metal has the property that inside the superconductor, both  $\vec{B}$  and  $\vec{E}$  are zero.

- a) For the  $\vec{E}$  field we found that the boundary condition at the surface of the superconductor is that  $\vec{E}$  must be normal to the surface, or in other words, that the tangential component must be zero. What is the analogous boundary condition for the magnetic field?
- b) A small current loop with its magnetic moment  $\vec{\mu}$  oriented at an angle  $\theta$  from the normal, is located a distance  $d$  from an infinite, superconducting sheet. Outline in a few sentences a method for finding the magnetic field everywhere, assuming that you already know what the field of a magnetic dipole is.
- c) Find the torque on the dipole as a function of angle. Deduce the equilibrium angles. Which are stable, which unstable?
- d) Find the force, as a function of angle, towards or away from the superconductor.



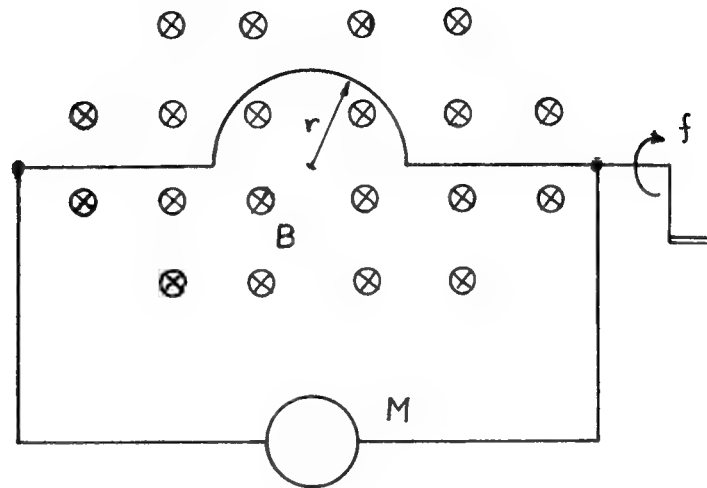
# CHAPTER 17

- 17-1. The figure below shows a uniform magnetic field  $\vec{B}$  confined to a cylindrical volume of radius  $r$ . The field  $\vec{B}$  is decreasing in magnitude at a constant rate of 100 gauss/sec. What is the instantaneous acceleration (direction and magnitude) experienced by an electron placed at  $P_1$ , at  $P_2$ , and at  $P_3$ ? Assume  $a$  is 5.0 cm.

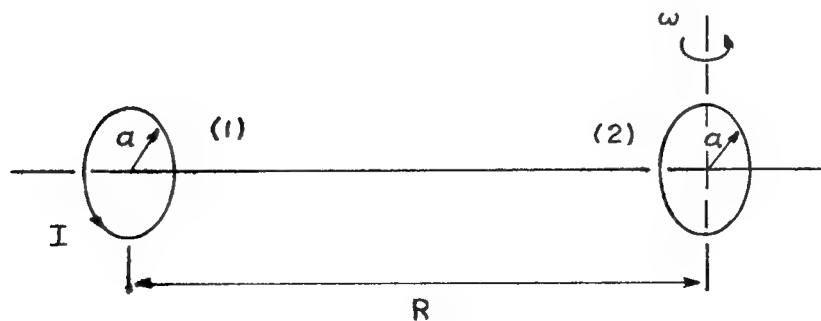


- 17-2. A stiff wire bent into a semicircle of radius  $r$  is rotated with a frequency  $f$  in a uniform magnetic field as shown in the figure. What are the amplitude and frequency of the induced voltage and of the induced current if the internal resistance of the meter  $M$  is  $R_M$  and the remainder of the circuit has negligible resistance? It must be assumed that the field produced by the current is small compared to the field  $\vec{B}$ . This requires the assumption that the current

is small or it will modify  $\vec{B}$ .



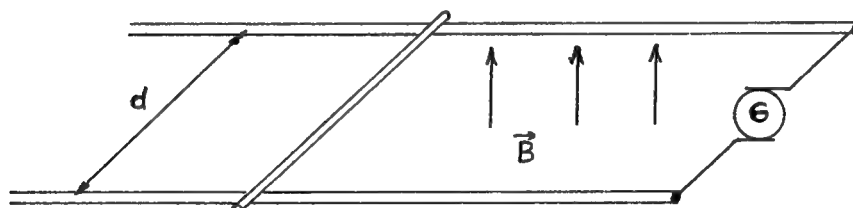
- 17-3. A small circular loop (1) of radius  $a$  carries the steady current  $I$ . Another loop (2), also of radius  $a$ , is placed on the axis through the center of loop (1) and at the distance  $R$ , with  $R \gg a$ . The planes of the two loops are parallel. Loop (2) is now rotated at the angular velocity  $\omega$  about one of its diameters. If the circuit of loop (2) is open so that no current flows in it, what is the emf generated?



17-4.

A metal wire of mass  $m$  slides without friction on two rails spaced a distance  $d$  apart, as in the figure. The track lies in a vertical uniform magnetic field  $\vec{B}$ .

- a) A constant current  $I$  flows from generator  $G$  along one rail, across the wire, and back down the other rail. Find the velocity (speed and direction) of the wire as a function of time, assuming it to be at rest at  $t = 0$ .
- b) The generator is replaced by a battery with constant emf  $\mathcal{E}$ . The velocity of wire now approaches a constant final value. What is this terminal speed? How as a function of time, does the speed approach this value?
- c) What is the current in part b) when the terminal speed has been reached? One must assume that the field due to the current  $I$  is much smaller than the field  $\vec{B}$ .



17-5.

A circuit contains two coils of inductances  $\mathcal{L}_1$  and  $\mathcal{L}_2$ , respectively, connected in series. These coils have mutual inductance  $\mathcal{M}$ . Find the resultant inductance of the circuit. If the wires to one coil are reversed how will the inductance change?

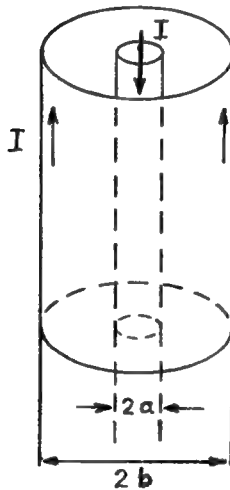


17-6.

A "coax" cable is made up of a wire which is surrounded by a coaxial conducting cylinder carrying the return current. The inner wire has radius a and the outer shell has radius b.

- a) Show that if the current in the inner wire is assumed to flow on the surface of the wire, the self-inductance per unit length of such a cable is

$$\mathcal{L} = \frac{1}{2\pi\epsilon_0 c^2} \ln \left( \frac{b}{a} \right) .$$



- b) Calculate the self inductance if the current is uniformly distributed throughout the central wire. Compare your results to see the importance of your assumption concerning the current distribution.

17-7.

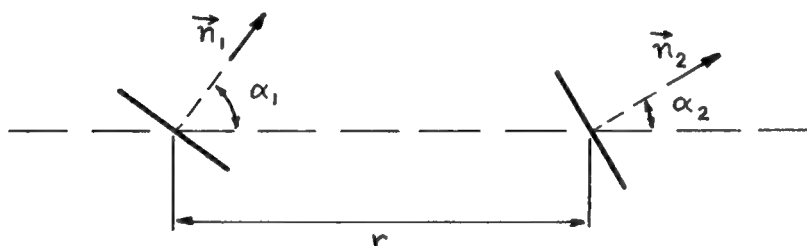
A toroidal coil of  $N$  turns has a square cross section, each side of the square being of length a, and has inner radius b.

- a) Show that the self inductance is

$$\mathcal{L} = \frac{N^2 a}{2\pi\epsilon_0 c^2} \ln \left( 1 + \frac{a}{b} \right)$$

- b) Express in similar terms the mutual inductance of the system formed by the coil and a long, straight wire along the axis of symmetry of the toroidal coil. Assume the conductors closing the circuit of which the long straight wire is a part are situated far from the coil, so that their influence may be neglected.
- c) Find the ratio of the self-inductance of the coil to the mutual inductance of the system.

17-8. Two plane current loops, each with area  $A$  and carrying current  $I$ , are placed a distance  $\underline{r}$  apart as shown below:

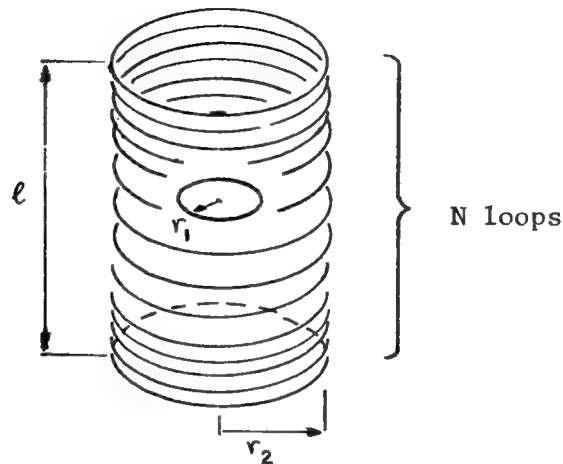


The normals to the current loops,  $\vec{n}_1$  and  $\vec{n}_2$ , make angles  $\alpha_1$  and  $\alpha_2$  with the line joining the loops. The vectors  $\vec{n}_1$ ,  $\vec{n}_2$  and the line joining the centers are coplanar.

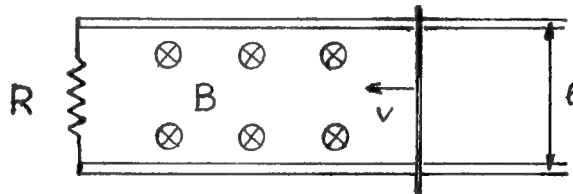
- a) Find the mutual inductance  $\mathcal{M}$  of this system of current loops. Assume the radius of each loop is much smaller than the distance between loops.
- b) Using this expression for  $\mathcal{M}$ , find the magnitude and direction of the force between the two loops.
- c) How would the force be different if the current were reversed in one or both of the current loops?

17-9. Consider a single loop coil of radius  $\underline{r}_1$  centered and coaxial with a solenoid coil of length  $\underline{l}$  made up of  $N$  loops of radius

$r_2$  where  $r_2 \gg r_1$ . If the single loop is called circuit 1 and the solenoid is called circuit 2, find both  $M_{12}$  and  $M_{21}$ .



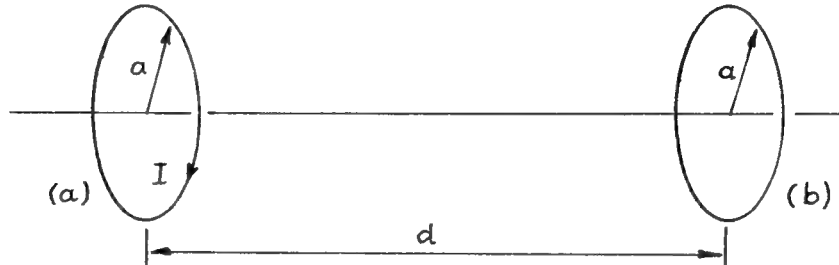
- 17-10. As shown in the sketch, a wire moves with a velocity  $\underline{v}$  while resting on two rails. There is a uniform  $B$  field perpendicular to the paper going into the paper.



If  $v = 100$  cm/sec,  $l = 10$  cm,  $B = 0.1$  w/m<sup>2</sup> and  $R = 10$  ohms, what is the current  $I$  through  $R$ ? (Neglect the field caused by the current). If the field arising from the induced current were not neglected, discuss if your estimate of the induced current would be larger, smaller or remain the same? If the magnet which is producing  $B$  is moving with the same velocity  $\underline{v}$  as the wire, does your answer change?

Is the self-inductance of the circuit around which the induced current flows increasing, decreasing, or unchanging with time?

- 17-11. Consider two coaxial loops of radius  $a$  which are separated by a distance  $d$ ;  $d \gg a$ . A current  $I = K_0 t^2$  is sent through one coil (coil a) as shown and the resistance of the second coil (coil b) is  $R$



- Neglecting self-inductance, what is the torque on loop b?
- Show that, if the self-inductance is neglected, the force on the loop b is:

$$\frac{24 \pi^4 a^8 K_0^2 t^3}{(4\pi\epsilon_0 c^2)^2 d^7 R}$$

In what direction is the force?

- In what way (qualitatively) is the true force and torque different from your estimate; i.e., how does the self-inductance affect the torque and force?
- Explain what would happen to parts a) and b) if the loop b were rotated  $90^\circ$  about an axis normal to the common axis of the loops.

## CHAPTER 20

20-1. It is sometimes convenient to consider complex solutions of differential equations.

- a) If we assume the fields vary sinusoidally in time and with the coordinate  $x$  (no  $y$  or  $z$  dependence), show that each component of, for example,

$$\vec{E} = \vec{E}_0 e^{+i(\omega t - kx)}$$

satisfies the wave equation. (Remember that the actual field is found by taking the real part of this expression.)

- b) Convince yourself that the real part of  $\vec{E}$  corresponds to a plane wave travelling along the  $x$ -axis. In what direction is it going?
- c) Show that the operation  $\vec{\nabla}$  when applied to a function like that in part a) becomes:

$$\vec{\nabla} = \vec{e}_x \frac{\partial}{\partial x} = \vec{e}_x (-ik)$$

where  $\vec{e}_x$  is a unit vector along the  $x$ -axis and  $i$  is  $\sqrt{-1}$ ; i.e., show that we can replace the  $\vec{\nabla}$  operation by a simple multiplication. What similar statement can you make about the time derivative?

- d) Using the results of c), write down (by inspection) how Maxwell's equations appear when applied to fields which vary sinusoidally with  $x$  and  $t$ . What relationship must exist between  $k$  and  $\omega$ ?
- e) If the field has the form  $\vec{E} = \vec{E}_0 e^{+i(\omega t + kx)}$  how do your answers change?

- 20-2. A plane electromagnetic wave of frequency  $\omega$  is reflected from a mirror travelling with a velocity  $\underline{v}$  in the same direction as the wave. Using Maxwell's equations calculate the frequency of the reflected wave as seen by a stationary observer. Compare this result with that obtained in Volume 1 using relativity per se.

## CHAPTER 21

- 21-1. Carry out the details of deriving Eq. (21.25).
- 21-2. Equation (21.1) gives an equation for calculating the electric field due to moving charges. Consider a dipole made up of a positive and negative charge oscillating about the origin along the z-axis; i.e., the motion of the positive charge is  $z_+ = \frac{d}{2} \cos \omega t$  and that of the negative charge is  $z_- = -\frac{d}{2} \cos \omega t$ . The dipole moment is then defined to be  $\vec{p} = d \cos \omega t \vec{e}_z$ . Show that the equation mentioned above can be used to calculate the entire electric field of the dipole:

$$E_{\phi} = 0$$

$$E_{\theta} = \frac{p}{4\pi\epsilon_0} \sin \theta \left[ \left( -\frac{\omega^2}{c^2 r} + \frac{1}{r^3} \right) \cos \omega \left( t - \frac{r}{c} \right) - \frac{\omega}{cr^2} \sin \omega \left( t - \frac{r}{c} \right) \right]$$

$$E_r = \frac{2p}{4\pi\epsilon_0} \cos \theta \left[ \frac{1}{r^3} \cos \omega \left( t - \frac{r}{c} \right) - \frac{\omega}{cr^2} \sin \omega \left( t - \frac{r}{c} \right) \right]$$

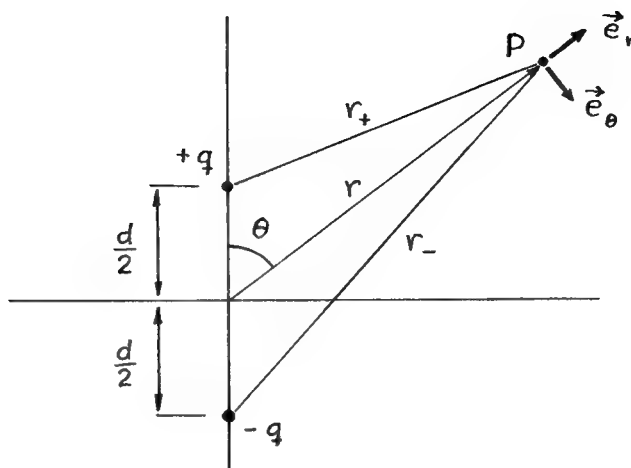
Assume the point P is at a distance  $r \gg d$  from the dipole.

Hints:

$$\vec{e}_{r_+} \approx + \vec{e}_r$$

$$\frac{d}{dt} (\vec{e}_{r_+}) \text{ and } \frac{d^2}{dt^2} (\vec{e}_{r_+}) \text{ are vectors nearly in the } \vec{e}_{\theta} \text{ direction}$$

(See figure on next page)



- 21-3. From the symmetry of Maxwell's equations and the form of the electric and magnetic field of an oscillating electric dipole,\* deduce the field of an oscillating magnetic dipole. The near field must resemble the field of a dipole formed by a small current loop of radius  $a$  ( $a \ll \frac{c}{\omega}$ ), and current  $i = i_0 \cos(\omega t)$

Answer:  $B_\varphi = 0$

$$B_\theta = \frac{1}{c} (E_\theta \text{ electric dipole})$$

$$B_r = \frac{1}{c} (E_r \text{ electric dipole})$$

$$E_\varphi = - (B_\varphi \text{ electric dipole})$$

$$E_\theta = E_r = 0$$

and replace  $p$  by  $\mu$  where  $\mu = \pi a^2 i_0$

---

\*

The electric fields of an electric dipole were given in problem 2; the corresponding magnetic field is found using Eq. (21.1)

$$c\vec{B} = \vec{e}_r \times \vec{E}$$



- 21-4. In problem 2, an oscillating dipole was made up of two moving charges. Another way of producing a dipole is as follows: the dipole is made up of two conducting balls joined by a wire of length  $d$ . An oscillating current in the wire is set up which establishes a net charge  $\pm q(t)$  at the ends, but leaves the wire neutral;  $q(t)$  can be represented as the real part of  $Q_0 e^{i\omega t}$ .

At any field point P, a distance  $r \gg d$  from the dipole, the integral for the retarded potential gives an exact expression for  $\varphi$ : (see figure below)

$$\varphi = \frac{Q_0}{4\pi\epsilon_0} \left[ \frac{\cos\omega(t - r_1/c)}{r_1} - \frac{\cos\omega(t - r_2/c)}{r_2} \right]$$

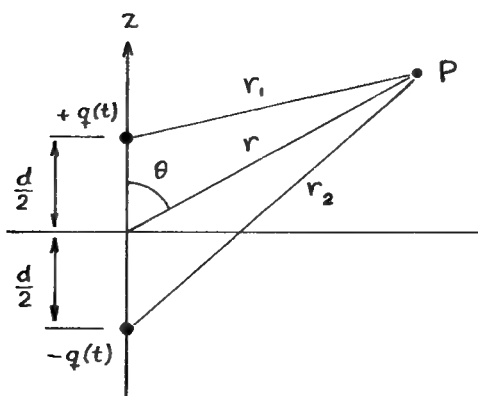
- a) Assuming that  $\frac{\omega d}{2c} \ll 1$  show that:

$$\varphi \approx \frac{Q_0 d \cos \theta}{4\pi\epsilon_0 r} \left[ \frac{1}{r} \cos \omega(t - r/c) - \frac{\omega}{c} \sin \omega(t - r/c) \right]$$

- b) Further show that

$$A_z \approx - \frac{Q_0 \omega d \sin \omega(t - r/c)}{4\pi\epsilon_0 c^2}$$

- c) Convince yourself that these potentials give the same electric and magnetic radiation fields (that part of the fields which is proportional to  $\frac{1}{r}$ ) as were found previously.



- 21-5. An antenna to be used at a frequency  $\omega = \frac{2\pi c}{\lambda}$  is made up of two colinear wires each one-quarter wave length long, and is driven at their junction by a sinusoidal voltage of the appropriate frequency. The resulting current distribution in the wires is to a high degree of approximation sinusoidal

$$i = -i_o \sin(\omega t) \cos\left(\frac{2\pi z}{\lambda}\right)$$

To find the radiation from this antenna it can be regarded as a superposition of many dipoles, each located at a point  $z$ , of length  $\Delta z$  with strength varying from dipole to dipole.

- a) Show that the proper dipole strength to use is

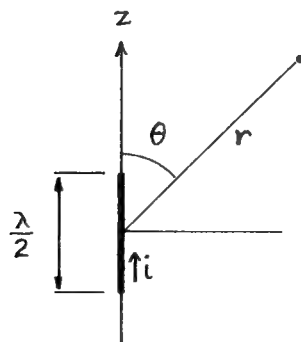
$$\Delta p = \left\{ \frac{i_o}{\omega} \cos \frac{2\pi z}{\lambda} \cos \omega t \right\} \Delta z$$

- b) Show that at large distances ( $r \gg \frac{c}{\omega}$ ), the field of the entire antenna is

$$E_\theta = \frac{2i_o}{4\pi\epsilon_0 cr} \frac{\cos\left(\frac{\pi}{2} \cos \theta\right)}{\sin \theta} \cos \omega\left(t - \frac{r}{c}\right)$$

$$B_\phi = \frac{1}{c} E_\theta$$

- c) Make a rough polar plot of  $E_\theta$  vs  $\theta$  for both this case and a single dipole and compare.



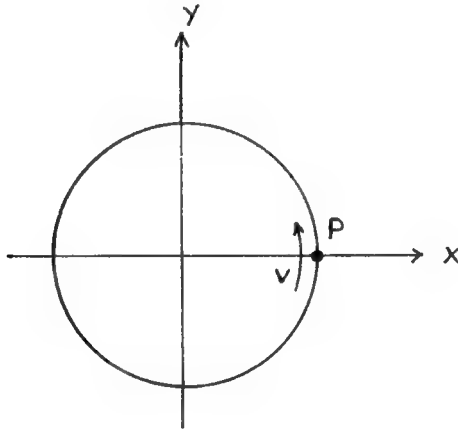
$$i = -i_o (\sin \omega t) \cos \frac{2\pi z}{\lambda}$$

$$\left(-\frac{\lambda}{4} < z < \frac{\lambda}{4}\right)$$

21-6. A particle with charge  $q$  moves in a circle of radius  $a$  with a uniform speed  $v$ .

- a) Find the scalar potential  $\phi$  at the center of the circle when the particle is at the point P.
- b) Find the vector potential  $A$  at the center at the same time.
- c) By calculation of the potential in the neighborhood of the center use Eq. (18.19) and (18.16) to determine the electric and magnetic fields at the center. What is the direction of the electric field with respect to the radius vector to point P?
- d) Also calculate these fields using Eq. (21.1).

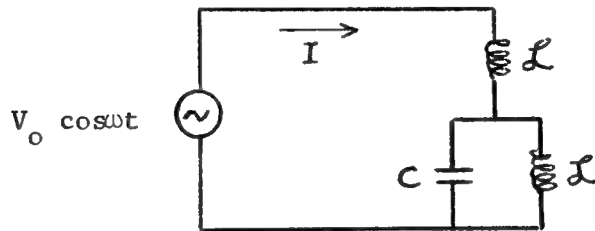
Note this problem is relativistic. The velocity  $v$  is not necessarily small compared to  $c$ .



## CHAPTER 22

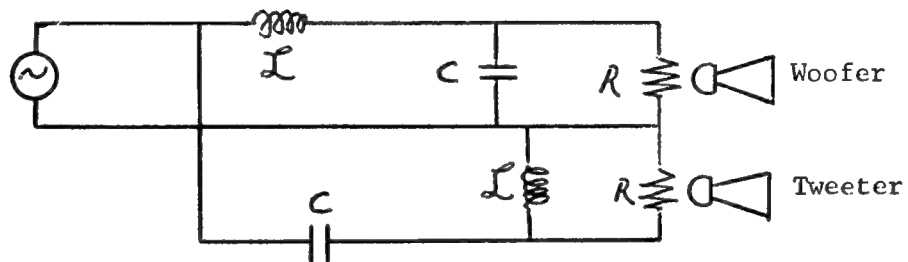
22-1. (Game) A network in the shape of a cubical frame has terminals at each vertex and 1 ohm resistances along each edge. Find the effective resistance between all possible sets of terminals.

22-2. a) Find the current  $I$  in the circuit shown below.



b) What is the current  $I$  if there is a mutual inductance  $M$  which couples the two inductances?

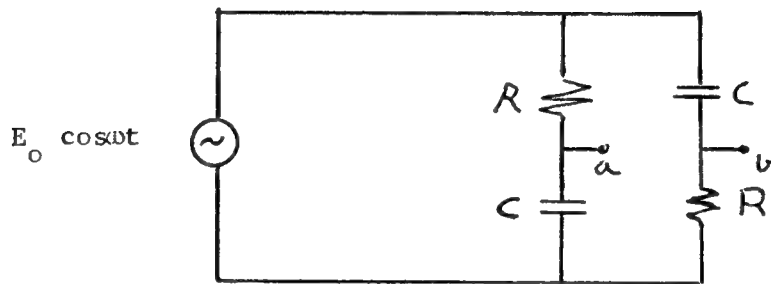
22-3. A crossover network for a hi-fi system is shown below



The effective resistance of each speaker is  $R$ .

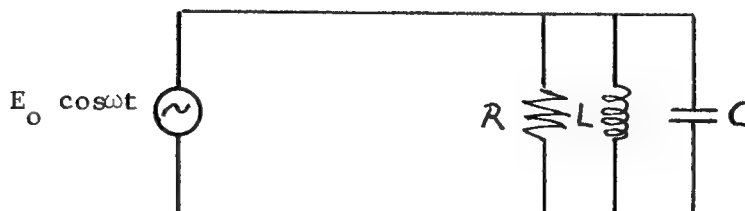
- a) Show that if  $R^2 = \frac{L}{2C}$ , the input impedance (the impedance seen by the generator) is purely resistive and equal to  $R$ .
- b) Show that the crossover frequency is given by  $\omega_c^2 = \frac{1}{LC}$ . The crossover frequency  $\omega_c$  is defined as the frequency for which each speaker receives one-half of the total power.

- 22-4. Show that for the circuit shown below the potential difference (voltage) from a to b has a magnitude that is independent of  $\omega$ . Draw a sketch of the phase of this potential difference as a function of  $\omega$ .

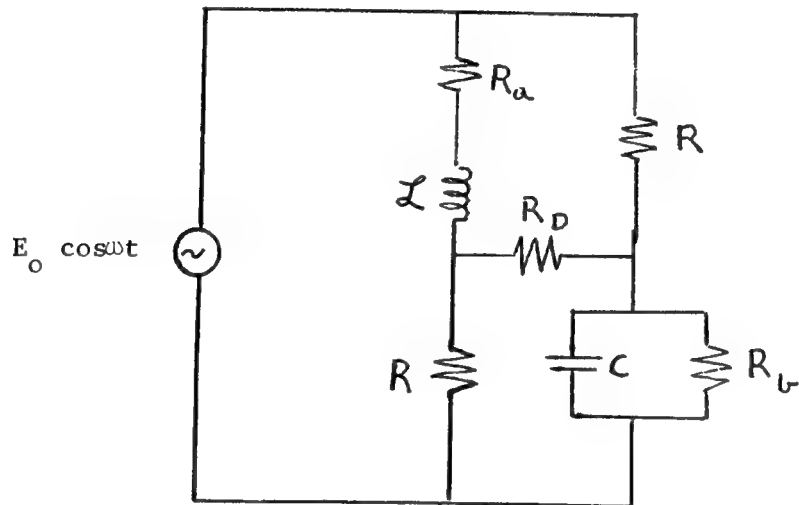


What would be the effect on the magnitude of the voltage across  $a - b$  and on its phase if the source had an internal resistance of  $R/10$ ?

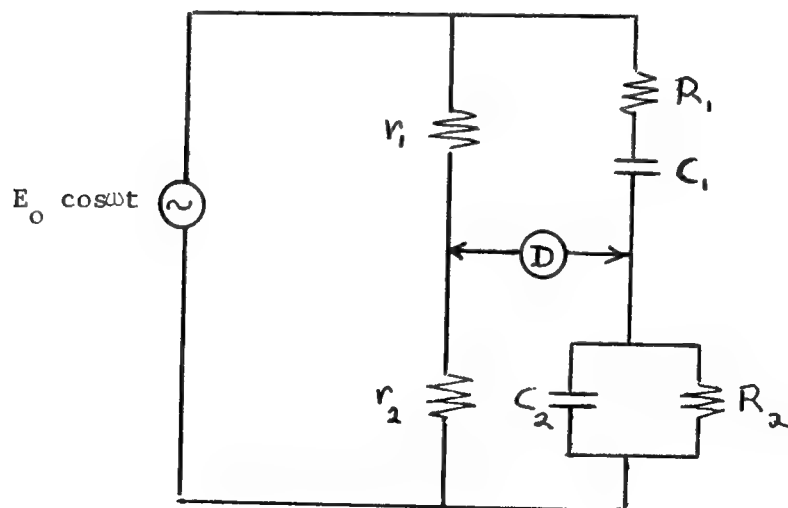
- 22-5. A simple parallel circuit is shown in the diagram below.
- Draw a rough graph showing the amplitude of the current as a function of frequency for selected values of  $L$ ,  $C$ , and  $R$ .
  - If  $R \gg \sqrt{\frac{L}{C}}$  compare the resonance frequency and the width of the resonance curve for this circuit to the same features of a circuit containing the same elements in series, but in which  $R \ll \sqrt{\frac{L}{C}}$ . Specifically consider the parallel circuit with  $R = K\sqrt{\frac{L}{C}}$  and the series circuit with  $R = \frac{1}{K}\sqrt{\frac{L}{C}}$ .



- 22-6. The circuit shown is a bridge used for measuring inductances. The generator has an alternating emf at the frequency  $\omega$ . When the bridge is balanced the current through the detector  $R_D$  is zero. Find  $\mathcal{L}$  in terms of  $R$  and  $C$ .



- 22-7. The circuit shown is a Wein bridge, frequently used in RC oscillators.

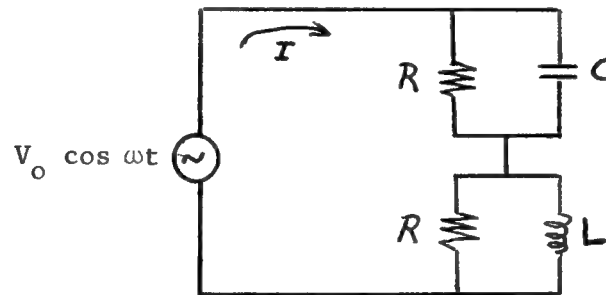


It is said to be balanced when no current flows through the detector. Show that balance requires that both of the following equations be simultaneously satisfied.

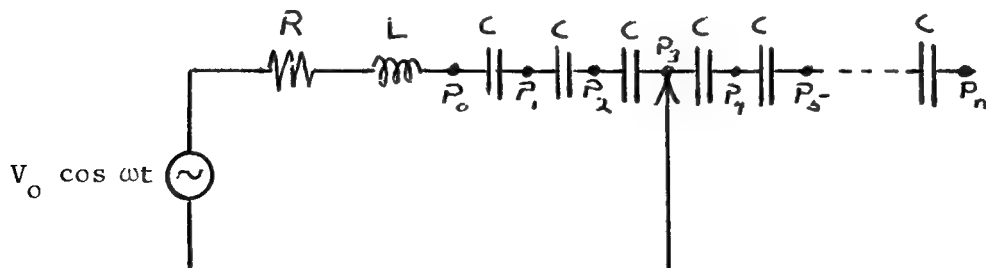
$$\left(\frac{r_1}{r_2}\right) = \left(\frac{R_1}{R_2}\right) + \left(\frac{C_2}{C_1}\right)$$

$$\omega = \frac{1}{\sqrt{R_1 R_2 C_1 C_2}}$$

- 22-8. A voltage source  $V(t) = V_0 \cos \omega t$  is applied to the circuit shown in the figure



- Show that if  $R$ ,  $L$ , and  $C$  are selected such that  $RC = L/R$  the current  $I$  is independent of frequency.
  - What is the phase difference between the applied voltage and the voltage across the capacitor-resistor pair (for  $RC = L/R$ )?
- 22-9. The circuit is arranged so that the connection to point  $P_3$  shown in the figure could be made to any of the points  $P_0, P_1, P_2, \dots, P_n$ .

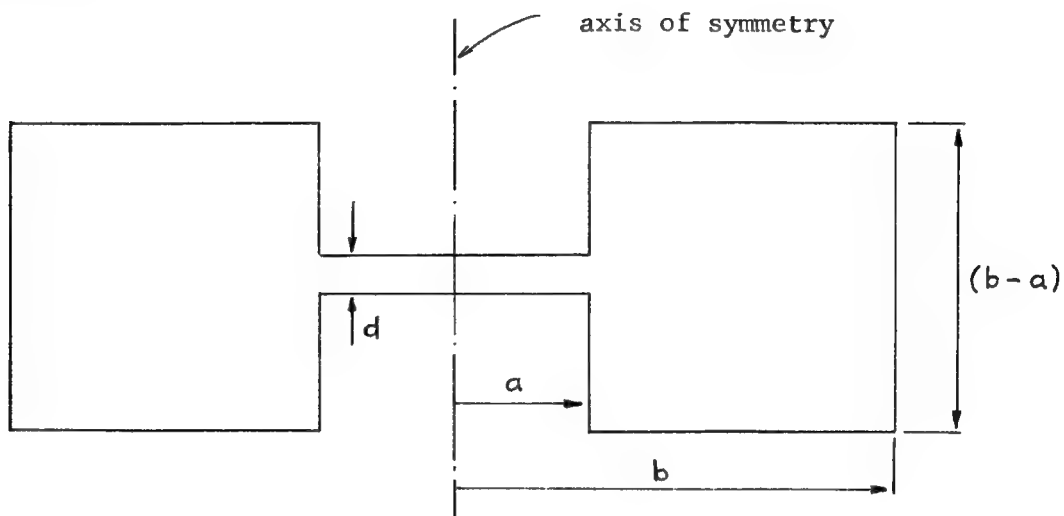


- a) Find an expression for the average power dissipated in  $R$  if the connection is made to  $P_m$ , where  $0 \leq m \leq n$ .
- b) If  $R = 1,000$  ohms,  $L = 10$  henries,  $C = 20 \mu\text{F}$ ,  $\omega = 100$  rad/sec:
- i) For what value of  $m$  is the power a maximum?
  - ii) For  $m = 2$  and  $V_o = 100$  volts, what is the maximum instantaneous voltage between  $P_o$  and  $P_2$ ? Across  $R$ ?



CHAPTER 23

- 23-1. Find the approximate resonant frequency of the cavity shown below. Assume  $d \ll a$ ,  $d \ll (b-a)$ . What are the main effects which you have neglected?



If the cavity is cooled uniformly (i.e., so the whole cavity is at the same temperature) does thermal contraction lead to an increase, a decrease, or no change in the resonant frequency?

## CHAPTER 24

- 24-1. A transmission line has inductance  $L_o$  per unit length and capacitance  $C_o$  per unit length. If the voltage and current are changing slowly (corresponding to transmission of signals with wavelength long compared to the line spacing), show that the governing equations are:

$$\frac{\partial V}{\partial x} = - L_o \frac{\partial I}{\partial t}$$

$$\frac{\partial I}{\partial x} = - C_o \frac{\partial V}{\partial t}$$

Hence show that  $I$  and  $V$  both satisfy the wave equation

$$\frac{\partial^2 I}{\partial x^2} = \frac{1}{v^2} \frac{\partial^2 I}{\partial t^2} \qquad \frac{\partial^2 V}{\partial x^2} = \frac{1}{v^2} \frac{\partial^2 V}{\partial t^2}$$

$$\text{where } v^2 = \frac{1}{L_o C_o}$$

Note the assumption regarding slowly varying signals is not necessary, but the proof of this is beyond the scope of this chapter.

- 24-2. The characteristic impedance of a transmission line is  $z_o = \sqrt{L_o/C_o}$  where  $L_o$  is the inductance per unit length and  $C_o$  is the capacitance per unit length.

Show that for a transmission line consisting of two thin strips of width  $\underline{b}$  and a distance  $\underline{a}$  apart ( $a \ll b$ )

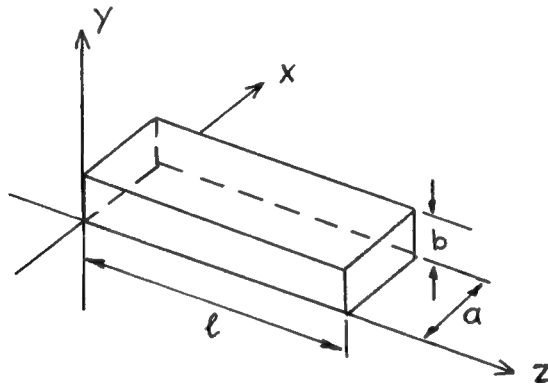
$$z_o \simeq \frac{1}{\epsilon_o c} \frac{a}{b}$$

- 24-3. A cavity is made by putting conducting plates across the ends of a section of a cylindrical coaxial line of length  $\ell$ .

- a) Find the frequency of the lowest mode for which the electric field is always radial.

- b) Give an expression for  $\vec{E}$ .
- c) Compare the resonant frequency to  $\omega_0 = 1/\sqrt{LC}$  where  $L$  and  $C$  are the inductance and capacitance of a length  $\ell$  of the coaxial line.

24-4. A rectangular waveguide made of perfectly conducting material has sides of length  $a$  and  $b$  as shown in the figure below.



The ends of a section of length  $\ell$  are covered with plates of conducting material; i.e., the waveguide is effectively a resonant cavity. If the electric field is given by the real part of:

$$\vec{E}(x, y, z, t) = E_0(x, z) e^{i\omega t} \vec{e}_y$$

what is  $E_0(x, z)$  for the cavity mode with the lowest resonant frequency? What is this lowest resonant frequency of the cavity?

24-5. A coaxial cable is composed of two concentric conducting cylinders. One end ( $x = 0$ ) is connected to a voltage generator which produces a voltage

$$V(t) = V_0 \cos \omega t \quad .$$

The other end of the cable ( $x = \ell$ ) is covered with a conducting plate. The inductance per unit length is  $L_0$  and the capacitance is  $C_0$ .

- If the length of the cable is  $\frac{5\pi c}{2\omega}$ , where  $c$  is the velocity of light, sketch the voltage between the conductors as a function of the distance  $x$ . Specify the values of  $x$  for which the voltage is maximum.
- Write an expression for the forward going and reflected traveling waves which make up the voltage across the conductors.
- What is the current at  $x = 0$ ,  $x = \frac{\ell}{2} = \frac{1}{2} \left( \frac{5\pi c}{2\omega} \right)$ , and  $x = \ell = \frac{5\pi c}{2\omega}$ ?
- If the voltage source is an ideal generator whose shaft turns with an angular velocity  $\omega$ , what average torque must be applied to the generator?

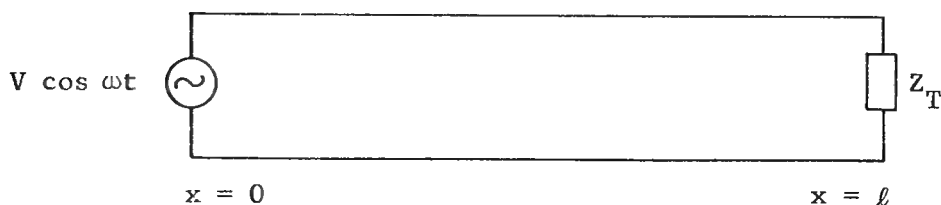
24-6. If a transmission line is terminated at  $x = \ell$  by an impedance  $Z_T$ , show that the "sending end" impedance ( $x = 0$ ) is given by

$$Z_s = iZ_0 \frac{\tan \omega \sqrt{LC} \ell - iZ_T/Z_0}{1 + iZ_T/Z_0 \tan \omega \sqrt{LC} \ell}$$

where  $Z_0 = \sqrt{\frac{L}{C}}$  is the characteristic impedance for the line.

What is  $Z_s$  if

- $Z_T = 0$
- $Z_T = \infty$
- $Z_T = Z_0$ ?



- 24-7. A transmission line with a characteristic impedance  $Z_1$  is connected to a transmission line with a characteristic impedance  $Z_2$ . If the system is being driven by a generator connected to the first line ( $Z_1$ ) show that the "reflection coefficient," i.e.,  $\frac{V_{\text{reflected}}}{V_{\text{incident}}}$ , is given by

$$\frac{V_{\text{reflected}}}{V_{\text{incident}}} = \frac{Z_2 - Z_1}{Z_2 + Z_1}$$

while the "transmission coefficient" is given by

$$\frac{V_{\text{transmitted}}}{V_{\text{incident}}} = \frac{2Z_2}{Z_1 + Z_2}$$

- 24-8. At JPL's Goldstone Tracking station the electronics cage is separated from the feed of the 85' receiver antenna by about 40 feet of wave guide. The inside dimensions of the wave guide are 5-3/4" by 11-1/2". If a 960 megacycle carrier is used, compare the signal velocity with the velocity in free space.
- 24-9. The electric fields inside of waveguides which are described in Chapter 24 have the property that the component of the electric field in the direction of propagation is zero; i.e., the electric field is transverse. (Modes of propagation such as these are therefore called TE, or transverse electric, modes.) There are also modes called TM modes in which there is no magnetic field in the direction of propagation. For the rectangular wave guide shown in Fig. 24-3 and 24-4, the vector potential of the TM modes is given by:

$$\vec{A} = \vec{e}_z \sin \frac{m\pi x}{a} \sin \frac{n\pi y}{b} e^{i(\omega t - k_z z)}$$

where

$$k_z = \sqrt{\left(\frac{\omega}{c}\right)^2 - \left(\frac{m\pi}{a}\right)^2 - \left(\frac{n\pi}{b}\right)^2}$$

- a) Satisfy yourself that the magnetic fields found from this are really transverse and show that the  $\vec{E}$  and  $\vec{B}$  fields satisfy the wave equation and the proper boundary conditions.  
Hint: We require that

$$\vec{E} = -\vec{\nabla}\phi - \frac{\partial \vec{A}}{\partial t} \quad \vec{B} = \vec{\nabla} \times \vec{A}$$

where

$$\vec{\nabla} \cdot \vec{A} = -\frac{1}{c^2} \frac{\partial \phi}{\partial t}$$

- b) Show that the nm mode is not propagated if

$$\omega < c \sqrt{\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2}$$

## CHAPTER 25

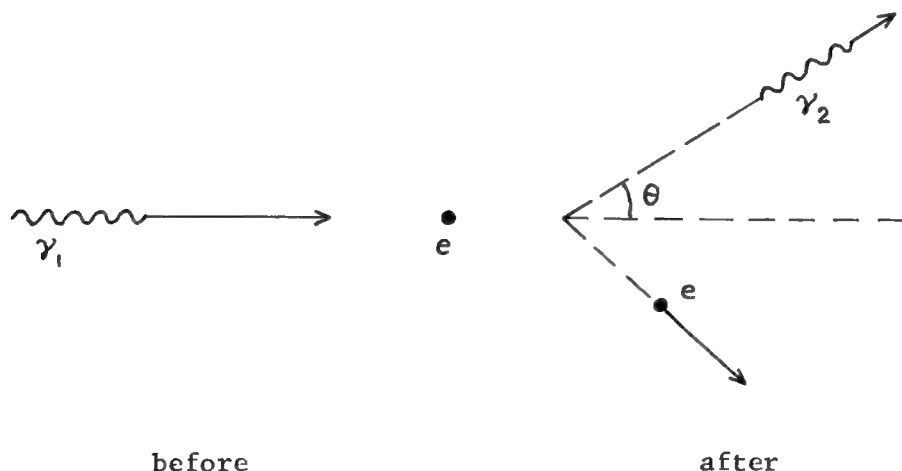
In the following problems the units are such that  $c = 1$ .

- 25-1. Write in four-vector form:

$$(\phi^2 - \vec{A}^2)$$

$$(\vec{A} \cdot \vec{j} - \rho\phi)$$

- 25-2. In the Compton effect, a stationary electron is hit by a photon, resulting in a change of momentum of each of them. Find the energy of the emitted photon in terms of its incident energy and the angle of deviation from its initial path.



- 25-3. A positron can be made by bombarding a stationary electron with a photon:

$$\gamma + e^- \rightarrow e^- + e^+ + e^-$$

What is the minimum photon energy? Use four-vectors and invariant combinations of them wherever possible.

- 25-4. An electron-positron pair can be produced by a photon ( $\gamma$ ) through the reaction

$$\gamma + e^- \rightarrow e^- + (e^+ + e^-)$$

It is impossible, however, for the reaction

$$\gamma \rightarrow e^+ + e^-$$

to occur for a single isolated photon even though the photon energy is larger than twice the electron rest mass and charge is conserved. Using four-vectors show that this is true.

- 25-5. A particle of mass  $m$  at rest is struck by another particle of mass  $M$  and momentum  $P$ . After a totally inelastic collision they coalesce to form a single new particle. What is its mass and velocity? Compare your results with the values that would be calculated non-relativistically.



## CHAPTER 26

In the following problems, the units are such that  $c = 1$ .

- 26-1. Write out and evaluate

$$\nabla_{\mu} F_{\mu\nu}$$

- 26-2. Find the four-vector whose three-vector part is

$$\rho \vec{E} + \vec{j} \times \vec{B}$$

What is the physical meaning of both time and space components of this four-vector?

- 26-3. Show that  $\vec{E}^2 - \vec{B}^2$  and  $(\vec{E} \cdot \vec{B})$  are invariant under Lorentz transformations. Note that if  $\vec{E}$  and  $\vec{B}$  form an acute angle in one frame, they do so in all frames. For what important physical phenomenon are both of these invariants equal to zero?

- 26-4. If  $\vec{E}$  and  $\vec{B}$  are the electric and magnetic fields at a certain point in space in a given frame of reference, determine the velocity of another frame in which the electric and magnetic fields will be parallel. There are many frames which have this property -- if we have found one of them then the same property will be had by any other frame moving relative to the first with a velocity parallel to the common direction of  $\vec{E}'$  and  $\vec{B}'$ . Therefore, we have a choice, and it is sufficient and convenient to find the frame which has a velocity perpendicular to both fields.

$$\text{Ans.} \quad \frac{\vec{v}}{1 + v^2} = \frac{\vec{E} \times \vec{B}}{E^2 + B^2}$$

- 26-5. In Chapter 26 the fields due to a charged particle moving with uniform velocity were obtained by the transformation of the potentials of a stationary charge to a moving frame. The fields  $\vec{E}$  and  $\vec{B}$  were obtained from  $A_\mu$  in the usual way.

Now, find the fields by starting with the fields from a stationary charge and using the transformation laws of the fields.

- 26-6. Show that the electric and magnetic fields of a charge moving with uniform velocity  $\vec{v}$  can be written

$$\vec{E} = \frac{q\vec{r}}{4\pi\epsilon_0 r^3} \frac{1 - v^2}{(1 - v^2 \sin^2\theta)^{3/2}}$$

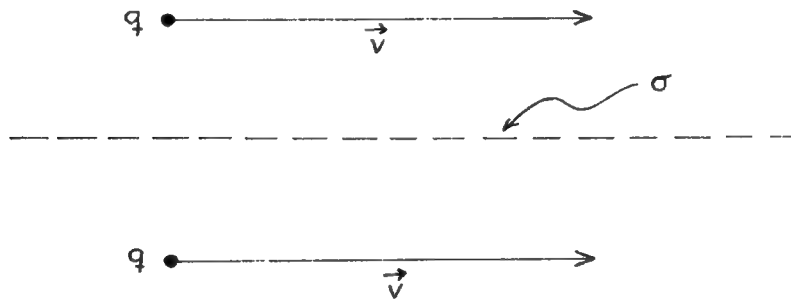
$$\vec{B} = \frac{q}{4\pi\epsilon_0} \frac{\vec{v} \times \vec{r}}{r^3} \frac{1 - v^2}{(1 - v^2 \sin^2\theta)^{3/2}}$$

where  $\vec{r}$  is the radius vector from the present position of the particle to the observer and  $\theta$  is the angle between  $\vec{r}$  and  $\vec{v}$ .

- 26-7. A very long straight wire carries a current  $I$  produced by electrons moving at speed  $\underline{v}$ . Stationary positive ions in the wire make the total charge density vanish.
- Find the fields outside the wire in a frame stationary with respect to the wire.
  - Transform the fields to a frame moving with the electrons. (In Chapter 13 the electric field observed from this moving frame was obtained by another method; Eq. (13.28)).
- 26-8. Two electrons with equal velocities  $\vec{v}$  are moving side by side the distance  $\underline{a}$  apart. Midway between them is an infinite sheet of

fixed positive charges with a surface charge density  $\sigma$ .

- a) How large must  $\sigma$  be in order that the electrons maintain the separation  $\underline{a}$ ?
- b) Compare the charge density needed if the electrons have an energy of 500 Mev to that needed if they are moving at a very low velocity.

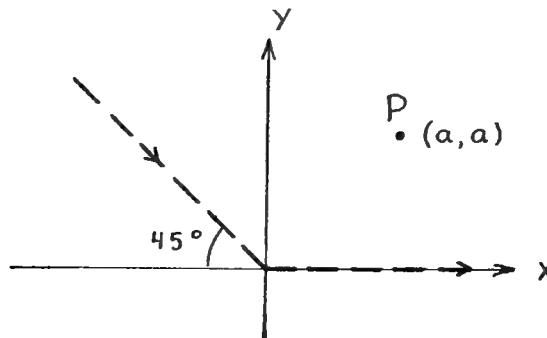


- 26-9. If  $f_\mu$  is the four vector force acting on a particle, and  $u_\mu$  is the four-vector velocity, show that

$$f_\mu u_\mu = 0$$

- 26-10. A particle of charge  $q$  moves in the  $x$ - $y$  plane at constant speed  $\underline{v}$ , along the trajectory shown by the dashed line in the figure. (It scatters at the origin). The speed remains constant throughout. At  $t = t_1$  it is at  $x = a$ ,  $y = 0$ .

- a) The point  $P$  is at  $x = y = a$ . Find the electric field at  $P$  at time  $t_1$  if  $v/c = 0.5$  ( $c$  is velocity of light).
- b) If in part a) the particle trajectory before the scattering were down the  $y$ -axis, how would your answer change?



## CHAPTER 27

- 27-1. Using the technique used to derive Eq. (27.11), find equivalent expressions for

$$\vec{\nabla} \times (\vec{A} \times \vec{B})$$

$$\vec{\nabla} (\vec{A} \cdot \vec{B})$$

- 27-2. How many megatons of energy are contained in the magnetic field of the earth external to the earth? Assume that the earth's field is a dipole with a strength of about  $2/3$  gauss at the equator. A megaton is the energy released by the explosion of 1-million tons of TNT or  $4.2 \times 10^{15}$  joules. In view of your answer, consider how much you think a one megaton hydrogen bomb exploded high in the atmosphere would disturb the earth's field.
- 27-3. For a long straight wire of resistance  $R$  per unit length, calculate the flux of  $\vec{S}$  at the surface of the wire when the wire carries a current  $I$ . Compare this with the heating calculated using Ohm's law.
- 27-4. A long coaxial cable is made of two perfectly conducting concentric cylinders. One end of the cable is connected to a battery whose terminal voltage is  $V$ , and the other end is connected to a resistance  $R$  so that there is a current  $I = V/R$ . Compute, using the Poynting vector, the rate of energy flow.
- 27-5. The average power radiated by a broadcasting station is 10 kilowatts.
- a) What is the magnitude of the Poynting vector at points on the surface of the earth 10 km distant? At this distance, the waves can be considered plane. It is reasonable to assume that the power is radiated by a  $1/4\lambda$  antenna above a perfectly conducting plane.
  - b) Find the maximum electric and magnetic intensities.

- 27-6. The fields of the lowest TE mode of the rectangular wave guide shown in Fig. 24-6 are given by:

$$\begin{aligned}\vec{E} &= \vec{e}_y E_0 \sin \frac{\pi x}{a} \cos (\omega t - k_z z) \\ \vec{B} &= -\vec{e}_x \cdot E_0 \frac{k_z}{\omega} \sin \frac{\pi x}{a} \cos (\omega t - k_z z) \\ &\quad - \vec{e}_z E_0 \frac{\pi}{\omega a} \cos \frac{\pi x}{a} \sin (\omega t - k_z z)\end{aligned}$$

- a) Show that the solution given above satisfies the boundary conditions for the problem.
  - b) Calculate the Poynting vector  $\vec{S}$  and the energy density  $U$ .
  - c) Calculate the average rate of energy flow across any plane perpendicular to the  $z$ -axis.
  - d) Calculate the average energy density in the wave guide.
  - e) Use the results of c) and d) to calculate the average velocity with which the energy is propagated. Show that this result is the same as the group velocity (24.27).
- 27-7. a) Find the rate of energy flow per unit area from an oscillating dipole with a dipole moment  $\vec{p} \cos \omega t$ .  
Hint: Keep only the radiation terms (i.e., those which drop off as  $1/r$ ).
- b) By integrating over the area of a large sphere centered on the dipole, show that the average power radiated is

$$\frac{1}{3} \frac{p^2}{(4\pi\epsilon_0 c^2)} \frac{\omega^4}{c}$$

- 27-8. A plane light wave is incident upon a free electron. The electron oscillates under the influence of the  $\vec{E}$  field. Calculate the ratio

of the energy radiated per unit time by the electron to the light energy incident per unit area per unit time. Assume that the light wave is of low frequency and neglect the effect of the  $\vec{B}$  field of the wave on the electron.

- 27-9. A dust particle in the solar system experiences two forces: the gravitational force of the sun and the planets, and the radiation force of light directed away from the sun. Since the gravity force is proportional to the volume of the particle and the radiation force is proportional to its cross section area, there will be a particle size for which these two forces are balanced. Assuming a spherical dust particle which absorbs all the radiation incident upon it, find the radius for which the forces balance.

An explanation for why a comet's tail points away from the sun has been based on the above phenomenon, assuming that the tail consists of small particles, perhaps even gas molecules. Is it a reasonable theory?

The energy radiated by the sun is  $4 \times 10^{26}$  watts, its mass is  $2 \times 10^{30}$  kg.

- 27-10. An "air-core" toroid of mean radius  $R$  and cross-sectional area  $\pi r^2$  is wound with  $N$  turns of wire [ $r \ll R$ ].

A current with the time dependence  $I(t) = Kt$  is turned on at  $t = 0$ .

- a) Directly from the magnetic field, find the energy stored in the magnetic field at time  $t$ .
- b) Find the direction and magnitude of the Poynting vector at a point just inside the toroid at time  $t$ .
- c) Using the Poynting vector, find the rate of change of the field energy inside the toroid at time  $t$ . Check your answer with that of part a).

## CHAPTER 28

- 28-1. If the rest mass of the electron is identified with the electrostatic energy of its charge and if the charge is uniformly distributed in the volume of a sphere, calculate the radius. Compare with the result given by Eq. (28.2).
- 28-2. It is well known that an electron in addition to charge and mass has an angular momentum (spin) and a magnetic moment related according to

$$\frac{\text{angular momentum}}{\text{magnetic moment}} = \frac{m}{q}$$

This is correct to about 0.1 per cent. Suppose the mass is given by Eq. (28.4).

- a) Take a uniformly charged spherical shell with charge  $q$  and radius  $a$  and place a magnetic dipole of strength  $\mu$  at the center. Show that the angular momentum of the electromagnetic field is

$$L = \frac{2}{3} \frac{q\mu}{4\pi\epsilon_0 c^2} \frac{1}{a}$$

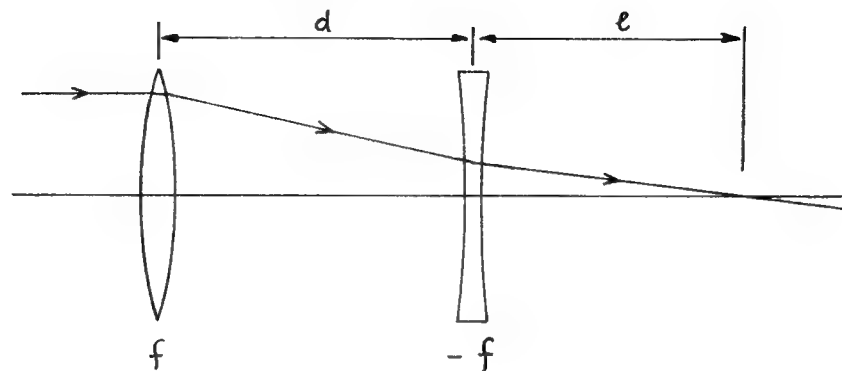
- b) Find the ratio of angular momentum to magnetic moment and compare with the value  $(\frac{m}{q})$  quoted above.
- c) Given that  $\mu_z$  for an electron is  $(\hbar q/2m)$ , calculate the maximum surface velocity of the spinning electron to give this magnetic moment. Make any comment you feel suitable. The quantity  $(4\pi\epsilon_0 c\hbar/q^2) = 1/\alpha$  has the value 137.

## CHAPTER 29

- 29-1. A charged particle (charge  $q$ , rest mass  $m_0$ ) is initially at rest at the origin. It is acted upon by a constant electric field in the  $x$ -direction.
- a) Calculate the velocity and position as a function of time (relativistically).
  - b) If the particle has an initial velocity  $v_0$  in the  $y$ -direction, how is your answer modified?
- 29-2. In a proton cyclotron the protons travel in circular paths in a uniform magnetic field. Find the "cyclotron frequency," the angular velocity as a function of  $q$ ,  $B$ ,  $m$  at low energies. How will the frequency change as the energy increases? At what energy has the frequency changed by 1 percent?
- 29-3. At  $t = 0$  a particle with mass  $m$ , charge  $q$  is located at the origin at rest. There is a uniform  $\vec{E}$  field in the  $y$ -direction and a uniform  $\vec{B}$  field in the  $z$ -direction.
- a) Find the subsequent motion,  $x(t)$ ,  $y(t)$ ,  $z(t)$  assuming non-relativistic motion. What restriction on  $E$  and  $B$  does this imply?
  - b) Can you suggest what the relativistic motion would be like? What happens if  $E/B > c$ ?
  - c) If we put a plate in the  $xz$  plane at  $y = 0$  and another parallel one at  $y = d$  with potential difference  $V_0 = E \cdot d$  and apply a magnetic field parallel to the plates, we have what is called a magnetron. If electrons are emitted from the negative cathode essentially at rest, how strong must the magnetic field be so that the electrons can't reach the positive anode?



- 29-4. The principle of alternating-gradient focusing can be illustrated by the following optical analog.



Even though the lenses have equal focal lengths, the combination has a converging action under certain circumstances.

- For parallel incoming light, determine  $\ell$  as a function of  $d$ .
- Under what conditions is the image real or virtual?

## CHAPTER 32

- 32-1. Show that in a non-polar material, the square of the index of refraction at low frequencies is equal to the dielectric constant.
- 32-2. At a frequency of about 6 megacycles per second, the ionosphere becomes transparent. Estimate the electron density in the ionosphere using the free electron model.
- 32-3. An electric field applied to a metal is held constant for a long time, and then is suddenly turned off. Using the free electron model of a metal show that the relaxation time (i.e., the time for the drift velocity to drop to  $1/e$  of its initial value) is equal to  $2\tau$ , twice the mean time between collisions.
- 32-4. In a metal there are plane-wave solutions to Maxwell's equations with the form

$$E_x = E_o e^{i(\omega t - kz)}$$

where  $k$  is a complex number. For low frequencies

$$k = (1 - i) \sqrt{\frac{\omega}{2\epsilon_o c^2}}$$

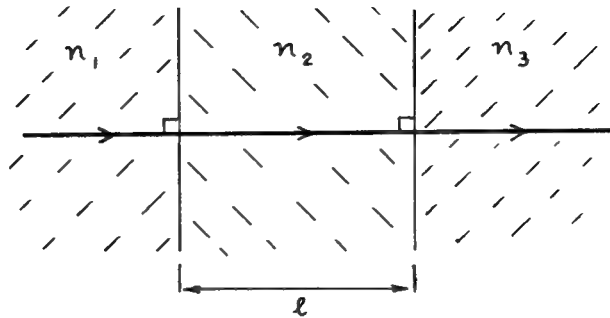
- a) Write an expression for the magnetic field associated with such a wave.
- b) What is the angle between  $\vec{E}$  and  $\vec{B}$ ?

- c) What is the ratio of the peak value of  $\vec{B}$  to the peak value of  $\vec{E}$  at any given value of  $z$ ?
- d) What is the phase difference between  $\vec{E}$  and  $\vec{B}$ ?  
(If the maximum of  $\vec{E}$  occurs at  $t_1$  and the maximum of  $\vec{B}$  occurs at  $t_2$ , the phase difference is defined as  $\pm \omega(t_1 - t_2)$ .)

32-5. Equation (32.50) suggests that the ultraviolet cut-off of a metal (i.e., the value of  $\omega$  at which  $\underline{n}$  changes from real to imaginary) is quite sharp. Experiments show that this cut-off is not sharply defined. Show by means of a better approximation for  $n^2$  that this experimental result is really in agreement with the theory.

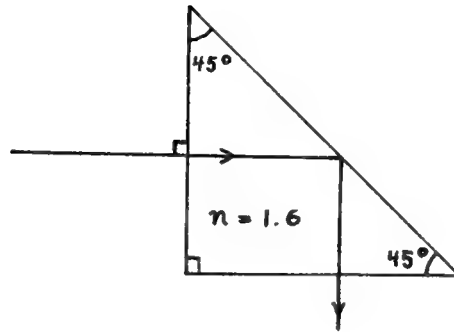
### CHAPTER 33

- 33-1. a) Determine the transmission coefficient for a plane electromagnetic wave passing through three dielectric media as shown below.



- b) Show that if  $n_2 = \sqrt{n_1 n_3}$  and  $l = \frac{\lambda_2}{4}$  the transmission ratio is unity. (This is the reason for "coating" lenses in good cameras and binoculars.)
- c) In binoculars to be used with ordinary white light, what is the thickness  $l$  of the coating?
- d) If it is only possible to coat one side of a lens, does it matter which side is coated? Why?
- 33-2. A beam of light with a wavelength  $4500 \text{ \AA}$  (in vacuum) is incident on a prism as shown in the figure below and totally reflected through  $90^\circ$ . The index of refraction of the prism is 1.6. Compute the distance beyond the long side of the prism at which the electric field strength is reduced to  $1/e$  of its value just at the surface. Assume the light is polarized so the  $\vec{E}$  is perpendicular to the plane of incidence. Is your answer changed if  $\vec{E}$  lies in the plane of incidence?

Figure for 33-2.



## CHAPTER 34

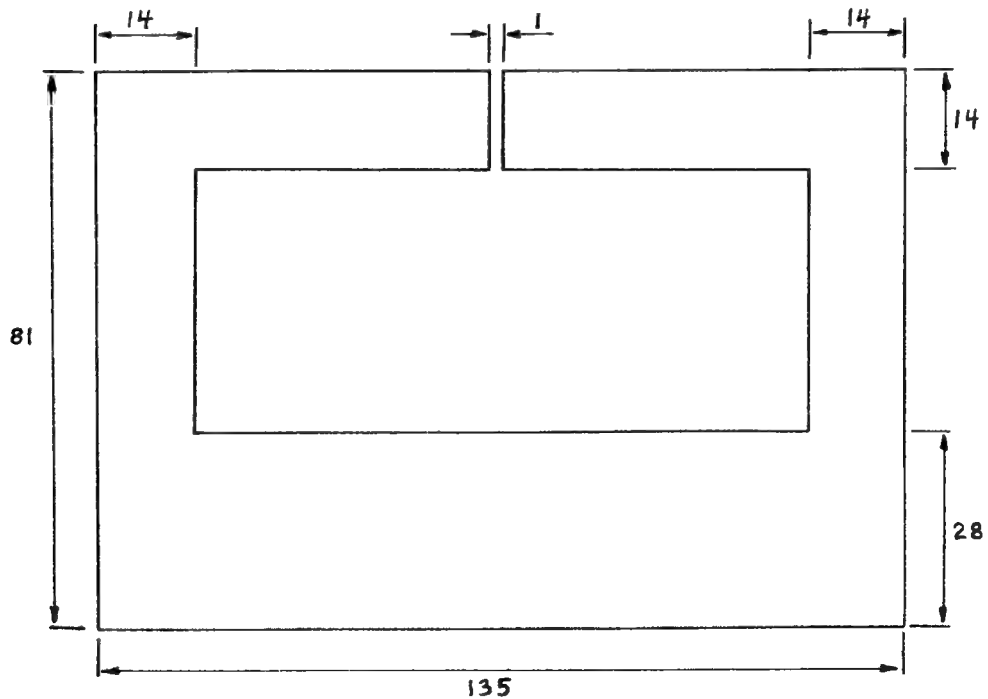
- 34-1. A charged particle moves in a plane perpendicular to a uniform magnetic field  $\vec{B}$ . Show that if  $\vec{B}$  changes slowly in time, the magnetic moment produced by the orbital motion remains constant. What do we mean by slowly?

## CHAPTER 35

- 35-1. In a low-energy cyclotron, protons go once around their circular orbit in a time  $T$  of about 0.13 microseconds. A proton nuclear magnetic resonance experiment with the protons in the same magnetic field shows a resonance at 21 megacycles per second. Find the  $g$ -value of the proton from these data.
- 35-2. Derive Eq. (35.9) in the manner suggested in the text. Can you reconcile this derivation with the proof in Chapter 34 that there can be no paramagnetism based strictly on classical physics?
- 35-3. A paramagnetic salt contains  $10^{22}$  ions per cc. with magnetic moment of 1 Bohr magneton. It is placed in a field of uniform magnetic induction of 10,000 gauss ( $1 \text{ weber/m}^2$ ).  
  
Calculate the percentage of excess parallel spins at both room temperature and at liquid helium temperature.
- 35-4. Derive an equation for quantum mechanical paramagnetism for particles of spin 1, following the derivation in Chapter 35 for spin 1/2.

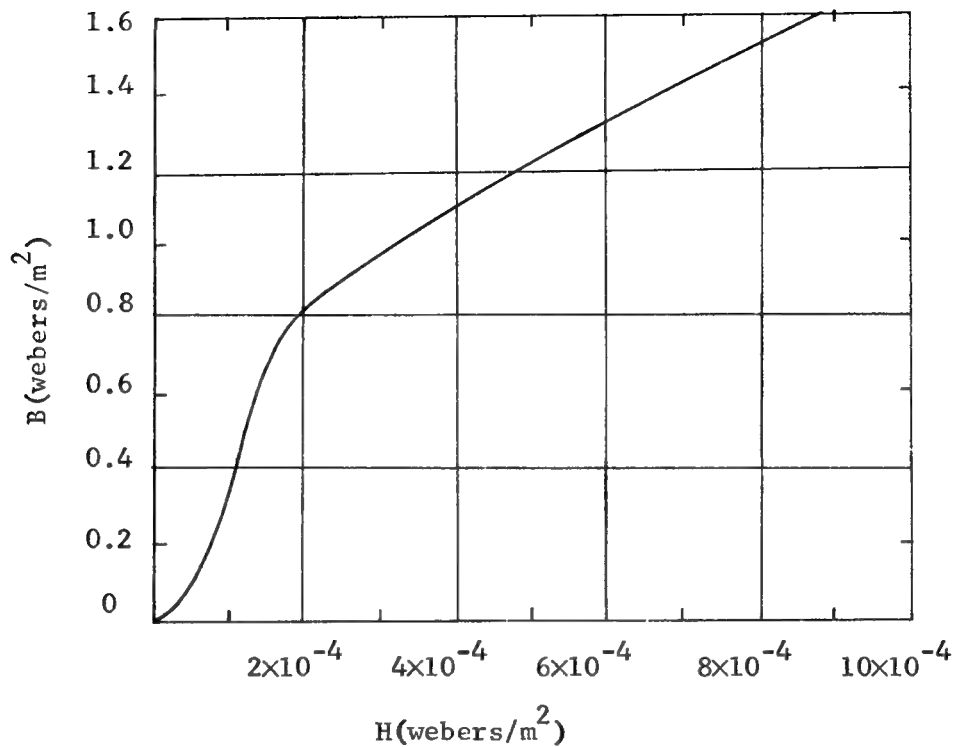
## CHAPTER 36

- 36-1. A uniformly magnetized sphere has a total magnetic moment of  $\frac{4}{3} \pi (a)^3 M$ , where  $a$  is the radius, and  $M$  the magnetization. Calculate the equivalent surface currents which can replace this sphere as far as external effects are concerned. Show that this current distribution has the same total magnetic moment.
- 36-2. The magnet frame shown below is wrapped by 2,150 turns of wire which carries a current of 5 amps. The iron has a uniform width (out of the paper) of 28 cm and has a  $B$  vs  $H$  curve as shown below. Make an estimate of how big a magnetic field will be obtained in the air gap. What are the main effects you have neglected? HINT: Since the  $B$ - $H$  curve is empirical and non-linear do not be surprised if the problem cannot be solved analytically or exactly.



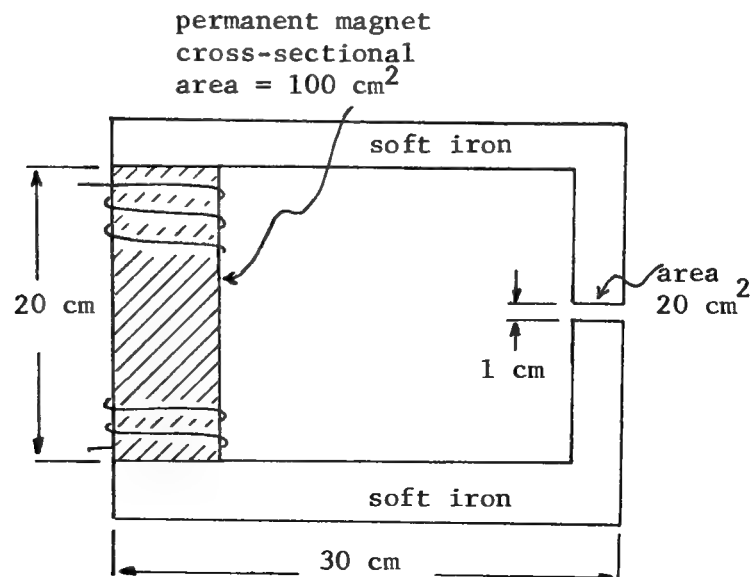
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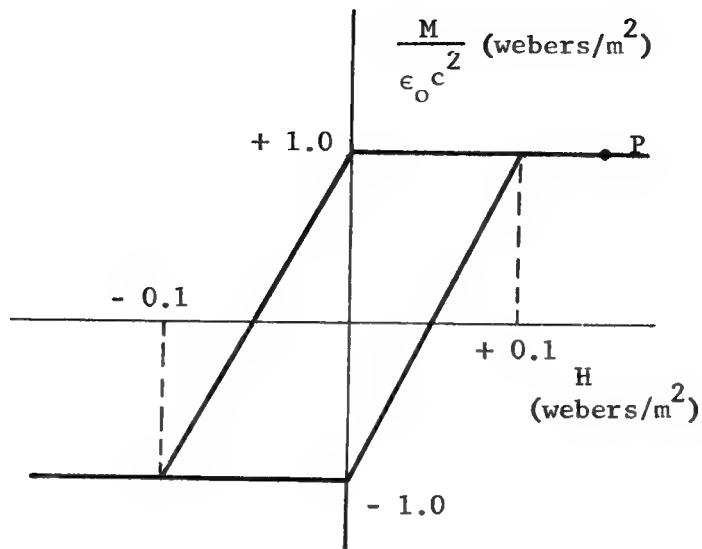




- 36-3. A magnetic flux is produced in an air gap by use of a bar of permanent magnet material and pole pieces of soft iron. The characteristics of the permanent magnet material are shown below.

The material is first magnetized to point P by passing a large current through an external coil. Assuming the soft iron to have infinite permeability and neglecting fringing of the flux, find the flux density in the gap after the current has been shut off.





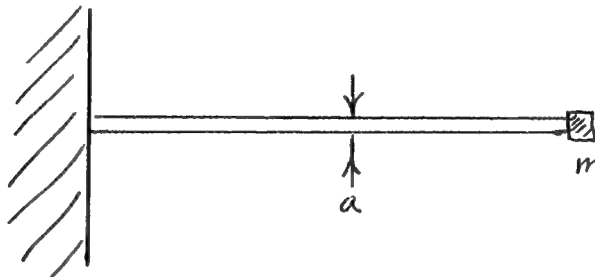
- 36-4. A very long cylindrical iron rod is permanently magnetized with a uniform magnetization  $\vec{M}$  which is pointing along the axis of the cylinder. Neglecting any end effects, find  $B$  and  $H$  in the iron. If there is a long needle-shaped cavity lined up along the axis, what is  $\vec{B}$  in the center of the cavity?

## CHAPTER 38

- 38-1. In many applications in space technology it is important to use materials which have a maximum strength to weight ratio.
- a) Compare the radii of a solid circular aluminum strut and a steel strut of equal stiffness and of equal length  $L$ . (The stiffness is defined as the ratio of the applied lateral force to the resultant displacement).
  - b) How do the masses of these struts compare?

- 38-2. An aluminum beam of length  $L$  with a square cross-section is held rigidly on one end as shown. A mass  $\underline{m}$  is attached to the free end of the rod.

Find the natural frequency of vibration of this system. Assume that the beam has a square cross-section with sides  $\underline{a}$ , that the mass of the rod is much smaller than  $\underline{m}$ , and that the mass  $\underline{m}$  can be considered a point mass.



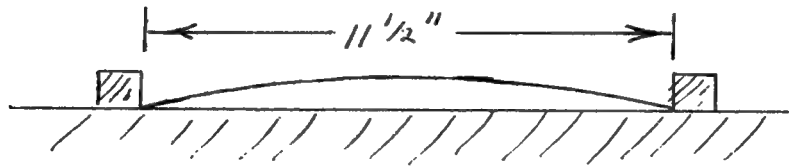
- 38-3. In Chap. I-47 the speed of sound in a fluid is found in terms of the rate of change of pressure with density. Show that for longitudinal waves in a solid (plane, compressional waves) the phase velocity is given by

$$v_{\text{long}}^2 = \frac{(1 - \sigma) Y}{(1 - 2\sigma)(1 + \sigma)\rho}$$

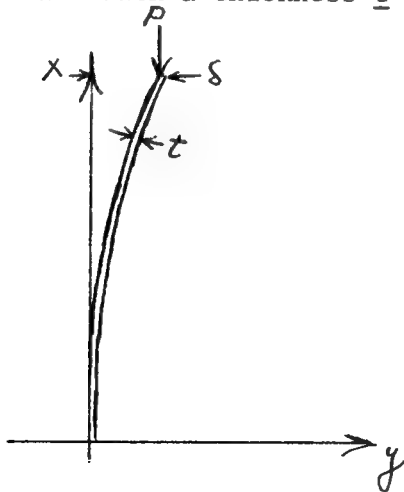
(This velocity applies to longitudinal waves in "infinite" medium. In this case the motion of each particle is always parallel to the direction of the wave; when the material is compressed by the wave there can be no sideways motion such as occurs in a rod which gets fatter when it is compressed.) How large do you think the dimensions of a block should be for this formula to be applicable.

- 38-4. A 12" steel ruler  $1/2$ " wide by  $1/32$ " thick is wedged between two blocks put on a table 11  $1/2$ " apart as shown in the figure below.

- Into what kind of a curve is the rule bent?
- Find the force against the blocks.



- 38-5. Determine the buckling load for a beam of length  $L$  clamped at one end and free at the other, as shown below. The beam has a rectangular cross-section with a thickness  $t$  and a width  $w$ .



## CHAPTER 40

- 40-1.      a) Prove to your own satisfaction the statement made in Chapter 40 that if a fluid cannot support a shear stress, the pressure is the same in any direction.
- b) As a mathematical exercise show the quite useful vector identity used in Chapter 40 that

$$(\vec{v} \cdot \vec{\nabla})\vec{v} = \frac{1}{2} \vec{\nabla}(\vec{v} \cdot \vec{v}) + (\vec{\Omega} \times \vec{v})$$

where  $\vec{\Omega} = (\vec{\nabla} \times \vec{v})$

- 40-2.      The liquid in a cylinder of circular cross-section rotates with a constant angular velocity  $\vec{\omega}$  about the axis. If the particles a distance  $\underline{r}$  from the axis rotate with a speed  $v = \omega r$ , find the shape of the surface at the top of the liquid.

Show that, as was pointed out in Chapter 40, the circulation per unit area, i.e.,  $\text{curl } \vec{v}$ , is twice the angular velocity at which the water is rotating.

- 40-3.      A sphere of radius  $\underline{a}$  and mass  $\underline{m}$  moves through "dry" water at a constant velocity  $\underline{v}$ . Show that the total kinetic energy of the ball plus the fluid is:

$$\frac{1}{2} \left( m + \frac{M}{2} \right) v^2$$

where  $M$  is the mass of the fluid displaced by the ball.

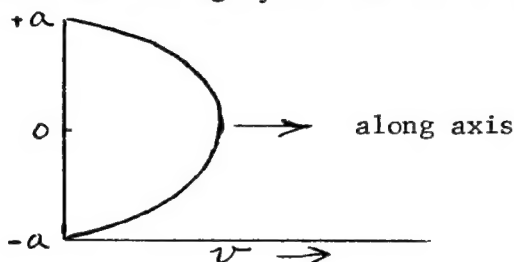
What is the total momentum of the ball plus the fluid?

## CHAPTER 41

- 41-1. If a ball of radius  $a$  is dragged with a constant velocity  $v$  through a viscous liquid slowly enough so that the flow is laminar, the force applied is a measure of the viscous force of the liquid on the ball.

Although you can figure out this force exactly, it is interesting to find the form of the force law from dimensional arguments after noting on what parameters the force should depend. Do so. Can you see by qualitative physical arguments why the parameters enter the way they do?

- 41-2. If a viscous fluid flows in a small pipe, the flow can be considered to be laminar, i.e., sheets of fluid in cylindrical tubes are flowing past each other. For a pipe of radius  $a$  the velocity profile across the pipe will look roughly as shown below.



Show that if  $r$  is the radial distance from the center of the pipe, if  $\eta$  is the viscosity of the fluid and if there is a pressure drop  $(P_1 - P_2)/L$  per unit length of pipe, the velocity is given by

$$v(r) = \frac{1}{4\eta} \frac{P_1 - P_2}{L} (a^2 - r^2) \quad .$$

In exact analogy to Ohm's law, the discharge rate of fluid  $Q$  from such a pipe can be related to the pressure differential  $\Delta P = P_1 - P_2$  by an equation

$$\Delta P = Q R$$

where  $\underline{R}$  is the "resistance" of the pipe. Find the resistance of a pipe of radius  $\underline{a}$  and length  $\underline{L}$ . Do you think this definition is just an exercise in words or can you think of reasons why analogies like this are useful. What is the analog to a condenser?

- 41-3. A large, shallow tray is partly filled with water (an "incompressible" liquid with viscosity  $\eta$ ). A thin flat sheet of wood floats on the water with its bottom surface at the height  $\underline{d}$  above the bottom of the tray. The other dimensions of the sheet are both much larger than  $\underline{d}$ . The sheet is moved horizontally at a slow speed  $\underline{v}$ . What is the rate of energy dissipation in a unit volume of the water near the middle of the sheet?



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